LETTERS TO THE EDITOR

A NOTE ON THE RELIABILITY OF TESTS*

Reliability and validity are two of the most important aspects of psychological tests (Intelligence Tests and Aptitude Tests, etc.). The most common method utilized to evaluate the reliability of tests is the split-half method.[1] The test is split into two halves in some way (*e.g.*, the odd items forming one half and the even ones the other half) and the coefficient of correlation between scores on them is worked out. This cœfficient is taken to be a measure of the reliability of the test, and is defined as the reliability-coefficient of the test.

But there is a certain amount of arbitrariness in this method, in the sense that the splitting can be effected in more than one way. Actually there are $n/2 \left(\frac{n}{2}\right)^2$ different ways of splitting a test of n items into two halves. Each of these ways of splitting the test gives its own estimate of the reliability-coefficient of the test, and there is no reason to suppose that each of these estimates must be the same. Hence for a test of n items the reliability coefficient worked out by one arbitrary splitting process is only one value chosen out of $n/2 \left(\frac{n}{2}\right)^2$ values lying near it.

To rectify this defect in the split half method Messrs. Kuder and Richardson[2] worked out a method which leads to unique values of the reliability-coefficient. Their formula is

$$r = \frac{n}{n-1}\left(\frac{\sigma^2 - \Sigma pq}{\sigma^2}\right)$$

where

r = reliability of the test
n = number of items in the test
σ = standard deviation of the test
p = proportion of subjects passing my item
q = I-p
Σ = Summation taken over all items in the test.

We can now put a lower limit to the value of the reliability-coefficient of tests. For, suppose there are 100 items in a test, and suppose at worst the scores lie between 50 and 98 (a pretty narrow range) ; then because of normal distribution of scores, the standard deviation σ can easily be assumed to be about 8. Further the greatest value of pq = ¼. Hence the Kuder-Richardson formula gives

$$r > \frac{100}{99}\left(1 - \frac{100}{4 \times 8 \times 8}\right),$$

which is $> 0 \cdot 5$.

Hence a test of fairly big size, whose reliability-coefficient is less than 0·5, is not good for use and must be rejected.

Although from theoretical considerations Kuder-Richardson formula is an improvement over the split-half method; yet its application in actual practice is a time-taking matter; for the term Σpq takes long to be evaluated. It would be welcome, therefore, if we could work out a formula which gives the reliability quickly even though approximately.

In the Kuder-Richardson formula let us replace $\frac{n}{n-1}$ by unity, and take $\sigma = \frac{n}{6}$. The value of pq lies between 0 and ¼, so that Σpq may be taken to be $\frac{n}{6}$. With these assumptions the reliability

$$r = 1 - \frac{n/6}{(n/6)^2} = \frac{\sigma - 1}{\sigma},$$

for $\frac{n}{6} = \sigma$.

This formula is independent of the term Σpq and is, therefore, very quick in giving the reliability of tests. It may be noted that bigger the size of the test, better will be the approximation given by this formula; for bigger is n, nearer to 1 is $\frac{n}{n-1}$.

For purposes of verification I have already applied the formula elsewhere[3] and found that the approximation given by this formula is correct upto the first place of decimal. Such an approximation is often quite satisfactory as far as reliability coefficients are concerned.

Ministry of Education, Government of India, Simla,
P. D. SHUKLA.
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1. Garret, H. E., *Statistics in Psychology and Education*, 1946, p. 313. 2. Kuder, G. F. and Richardson, M. W., The Theory of the Estimation of Test Reliability" *Psychometika*, 1937, **II**, **3**, 151–60. 3. Shukla, P. D., "Some Statistical Studies about Civil Selection Boards Abstract," *Proc. Indian Sci. Congress*, Delhi, 1947.

* This note was read in the Indian Science Congress Session at Patna, in January 1948.

While considering the Kuder-Richardson formula

$$r = \frac{n}{n-1}\left[\frac{\sigma^2 - \Sigma pq}{\sigma^2}\right] \qquad (1)$$

the author suggests that we can assume that $\frac{n}{n-1}$ approaches unity as n becomes larger and larger and also that we can replace σ by $\frac{n}{6}$ and $\Sigma pq = n/6$. The value of pq lies between 0 and 1/4 and the reasons for assuming $\Sigma pq = n/6$ is not very clear, apart from the arithmetical ease in the further simplification of (1). Since in (1) the term Σpq is subtracted, it is better if we assume the maximum value of $\Sigma pq = n/4$ and proceed as suggested in this paper. The value of r on simplification becomes the minimum and the formula (1) reduces to

$$r = \frac{\sigma - 1 \cdot 5}{\sigma}.$$

If this value of r can be assumed to be reliable, it will be so whatever be the values of p and q, while the simplifications suggested by the author will be reliable only when $0 < \Sigma pq < n/6$. Besides, this new formula has got all the advantages which is claimed by the author for his suggested formula.

M. C. S. AND N. S. N.
Bangalore,
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ON EINSTEIN'S GENERALIZED THEORY OF GRAVITATION

A PAPER, bearing the same title as the present communication, was recently communicated by us to the National Institute of Sciences of India. We have discussed in it the field equations of the generalized theory* and outlined a method of successive approximations for working out the interaction between a gravitational field and an electromagnetic field. The computation of interaction terms is an extremely laborious task and, so far as we know, has not been carried out yet. The results that are given below seem to be new and of considerable interest.

The theory is based on a Hermitian tensor g_{ik}:

$$g_{ik} = a_{ik} + \sqrt{-1}\, b_{ik},\ a_{ik} = a_{ki},\ b_{ik} = -b_{ki}. \qquad (1)$$

We consider a pure gravitational field as given by

$$a_{11} = a_{22} = a_{33} = -(1 + m/2r)^4, \quad a_{44} = (1 - m/2r)^2\,(1 + m/2r)^{-2},\ a_{ij} = 0,\ i \neq j \qquad (2)$$

and a pure electromagnetic field as given by

$$b_{13} = b_{34} = \phi,\ b_{12} = b_{14} = b_{23} = b_{24} = 0, \quad \phi \equiv A \cos \frac{2\pi}{\lambda}(x - t), \qquad (3)$$

in the usual notation. In the complex field unifying the two a_{ij} is modified by ϕ and b_{ij} by m. The exact nature of the interaction of the two fields is certainly very complicated. Our computations show that the usual Maxwell equations are modified as follows:

$$\begin{aligned} -\frac{\partial E_z}{\partial y} + \frac{\partial E_y}{\partial z} - \frac{\partial H_x}{\partial t} &= \frac{2\phi m z}{r^3}, \\ -\frac{\partial E_x}{\partial z} + \frac{\partial E_z}{\partial x} - \frac{\partial H_y}{\partial t} &= 0, \\ -\frac{\partial E_y}{\partial x} + \frac{\partial E_x}{\partial y} - \frac{\partial H_z}{\partial t} &= -\frac{2\phi m x}{r^3}, \\ \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} &= -\frac{2\phi m z}{r^3}. \end{aligned} \qquad (4)$$

The components of the electric current-and-density vector are found to satisfy, to the same degree of approximation, the following equations—

$$\begin{aligned} \Box\, \rho + \frac{12mxy}{r^5}\,\phi_1 &= 0, \\ \Box\, \sigma_x + \frac{12mxy}{r^5}\,\phi_1 &= 0, \\ \Box\, \sigma_y + \frac{12m}{r^5}(y^2 - z^2)\phi_1 &= 0, \\ \Box\, \sigma_z + \frac{24myz}{r^5}\,\phi_1 &= 0, \end{aligned} \qquad (5)$$