

This gives on integration

$$e^{\mu/2} \left(\mu'' - \frac{1}{2} \mu'^2 - \frac{1}{r} \mu' \right) = \phi(r) \quad (8)$$

while the second equation (6) gives

$$\mu = \int e^{\mu/2} f \frac{1}{2} dt + \psi(r) \quad (9)$$

(8) and (9) will together yield a relation of the form,

$$\log \phi(r) = \psi'' - \frac{1}{r} \psi' + \frac{1}{2} f r dr \int \frac{1}{r} \left(\psi'' - \frac{1}{r} \psi' \right) dr$$

$$\text{or, } \log \{ \phi(r) \} = \psi'' - \frac{1}{r} \psi' + \frac{1}{2} \psi \quad (10)$$

giving $\phi(r)$ in terms of $\psi(r)$ and its derivatives. It will be seen that

$$\phi(r) = 2\beta(r).$$

As regards the other points mentioned by Narlikar in his letter I shall not discuss them here as these will be incorporated in a separate paper to be communicated for publication elsewhere.

Poona,
May 28, 1947.

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1. Narlikar, *Curr. Sci.*, 1947, 16, 113. 2. Moghe, *Proc. Ind. Acad. Sci.*, 1939, 10, 407. 3. *Ibid.*, 1939, 10, 275.

ANOTHER METHOD OF CALCULATING SQUARES OF NUMBERS

In a previous article¹ I have given a new method of obtaining squares of numbers, large or small, by mental calculation only. With a view to reducing this mental work further, especially when a majority of figures in the given number are greater than 5, another method is illustrated here.

(1) A number of two digits—88.

$$\begin{array}{r} 88 \\ 12 \\ \hline 144 \\ 76 \\ \hline 7744 \end{array}$$

Subtract the units digit of the original number from 10, and the remaining ones from 9; thus the new number is 12. Square this number, 12, by the method last suggested.¹ It is 144. Double the original number, and enter the first two digits of this product beginning from below the third digit of the last number, as it is necessary to leave out as many digits as there are in the original number. Thus we get 76. Add the numbers together, as in the example, and we get 7744, the required square.

(2) A number of three digits—649.

$$\begin{array}{r} 649 \\ 351 \\ \hline 123201 \\ 298 \\ \hline 421201 \end{array}$$

Subtract the units digit of the original number from 10, and the remaining ones from 9,—351. Square 351—123201. Double the given number, and enter the first three digits of the product beginning from below the fourth digit

of the last number as shown in the example—298. Addition gives 421201, the square of 649.
(3) A number of five digits—75684.

$$\begin{array}{r} 75684 \\ 24316 \\ \hline 591267856 \\ 51368 \\ \hline 5728067856 \end{array}$$

Subtract the units digit of the original number from 10 and all the remaining digits from 9—24316. Square this number—591267856, which is written below the line. Double the original number, and write down the entire product, excepting the last digit, in such a way that the units digit of this comes exactly below the sixth digit of the square as shown above. Add together the two numbers between the lines,—5728067856, the required square.

The former method¹ gives the result quicker when the figures in the number are less than 5; whereas by this method the square is obtained more readily with numbers of more than 5 digits.

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April 9, 1947.

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1. Siddiqi, *Curr. Sci.*, 1947, 16, 178.

AN ADDITIONAL RAINBOW

At 6-10 p.m. on Monday, the 14th July 1947, there was a heavy shower of rain with the sun shining very bright in the west. Thinking that these were the most favourable conditions for the formation of an almost complete rainbow, I went out and saw, as expected, the usual beautiful rainbow, very intense in colour, forming nearly a full semicircle without any gap, accompanied by the secondary and the super-numeraries which were all explained by Airy on the basis of the theory of diffraction by rain drops.

But one surprising feature of this occasion was the appearance of an extra rainbow. Fig. 1 indicates the size and the disposition of the additional rainbow with respect to the usual one. The former is given in the thick line to differentiate it from the latter. Its general features were:

1. It was not concentric with the normal rainbow, there being a definite inclination between the two as indicated in the figure.
2. Its intensity was nearly equal to that of the usual secondary rainbow.
3. Only a small portion of it could be seen covering the arc of another circle included between the normal primary and secondary as given in the figure.
4. The portion thus seen was to the south, but its counterpart at the other end of the corresponding circle was almost merged with the northern bottom of the normal primary.