$$2F_1\left[\begin{matrix}\frac{5}{4}+m, \frac{5}{4}-m \\ \frac{7}{4}-k\end{matrix}; \frac{1}{2}\right] \int_0^\infty \frac{f(x)}{x}\,dx.$$

Theorem 2B. If $\phi^k{}_m(p)$ is the Whittaker Transform of $f(x)$, and $f(p)$ is the Laplace Transform of $\psi(x)$, then

$$\phi^k{}_m(p) = \frac{\Gamma(\frac{9}{4}+m)\,\Gamma(\frac{9}{4}-m)}{4\,p\,\Gamma(\frac{11}{4}-m)} \times$$

$$\int_0^\infty 2F_1\left[\begin{matrix}\frac{9}{4}+m, \frac{9}{4}-m \\ \frac{11}{4}-m\end{matrix}; -\frac{s-p}{2p}\right] \psi(s)\,ds.$$

Theorem 1 C. If $\phi^k{}_m(p)$ is the Whittaker Transform of $f(x)$, then

$$\sum_{r=0}^{\infty} \frac{1}{r\,!} \left(\frac{1}{\alpha}-1\right)^r \phi_m{}^{k+r}(p)$$

$$= p\,\alpha^k \int_0^\infty (2xp)^{-\frac{1}{2}}\, e^{xp(1-\alpha)}\; W_{k,m}(2xp\alpha)\, f(x)\,dx.$$

3. Theorems marked A are exactly similar in form to the corresponding ones in the theory of Operational Calculus, those marked B have generalised appearances and give, as particular cases, the corresponding theorems of Operational Calculus, due to Van der Pol[3] and Humbert.[4] It is interesting to note that the Theorem 1 C has no analogue in the Ordinary Operational Calculus.

4. Special cases, besides the ordinary Laplace's Transform, of the general transform (2) may be obtained by taking particular cases of Whittaker functions. The following may be noted:

(i) K_n – transform [$K_n(x)$ denoting Bessel Functions of the second kind for imaginary arguments]

$$\phi_m{}^0(p) = (p/\sqrt{\pi}) \int_0^\infty (2xp)^{\frac{1}{2}}\, K_m(xp)\, f(x)\,dx.$$

This is obtained by taking $k=0$ in (2).

(ii) $L^n{}_s$–transform [$L_s{}^n(x)$ denoting generalised Laguerre Polynomials].

$$\phi^{\frac{1}{2}+\frac{1}{2}n+s}_{\pm\frac{1}{2}n}(p) = (-)^s\, s\,!\, p \times$$

$$\int_0^\infty \{(2xp)^{\frac{1}{2}n+\frac{1}{2}}\, e^{-xp}\, L^n{}_s(2xp)\}\, f(x)\,dx.$$

This is obtained by taking $k=\frac{1}{2}+\frac{1}{2}n+s$ and $m=\pm\frac{1}{2}n$ in (2).

(iii) D_n – transform [$D_n(x)$ denoting Weber's parabolic cylinder functions]

$$\phi^{\frac{1}{2}n+\frac{1}{4}}_{\pm\frac{1}{4}}(p) = 2^{-\frac{1}{2}n}\, p \times$$

$$\int_0^\infty D_n\,(2\sqrt{xp})\, f(x)\,dx.$$

This is obtained by taking $k=\frac{1}{2}n+\frac{1}{4}$ and $m=\pm\frac{1}{4}$ in (2).

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1. See Widder, *The Laplace Transform* (Princeton University, 1943, and Meijer, *Proc. Kon. Akad. v. Wetensch, Amsterdam*, 1941, **44**, 727–37. 2. Whittaker and Watson, *Modern Analysis*, (fourth edition, 1935), 337–54. 3. Van der Pol, *Phil. Mag.*, 1929, **8**, 864–5. 4. Humbert, *Bulletin de la Soc. Mathematique*, 1937, **65**, 3.

A MODIFIED DEFINITION OF PROBABILITY

THE author of a recent paper[1] on the definition of probability points out that the Von Meyses' definition of probability, namely, that probability p is the limit of the sequence s/n as n tends to infinity, where s is the number of successes in n trials, imposes a restriction on the number of successes in m trials succeeding a certain number of n trials. If μ denotes the number of successes in m trials, namely, from $n+1$th to the $n+m$th, μ should lie between

$$(p-\epsilon)\,m - 2\,\epsilon n_0 \text{ and } (p+\epsilon)\,m + 2\,\epsilon n_0$$

for every $n \geq n_0$. This has been derived from the fact that $s/n \to p$ and $\frac{s+\mu}{n+m} \to p$. It is argued that since ϵ is a small quantity, the above restriction implies that μ should lie between the narrow limits $(p-\epsilon)m$ and $(p+\epsilon)m$; whereas μ can take any value from 0 to m.

It is surprising that the author has gone so far to get this restriction, which is obvious on the face of the definition itself. For mp denotes the expected number of successes in m trials, and when m is sufficiently large μ should lie between $m(p-\epsilon)$ and $m(p+\epsilon)$; this is true because p is the limit of μ/m as m tends to infinity. It is not advisable to consider the m trials made after the nth trial separately. Either they can be considered as two separate experiments or may be considered as one experiment of $n+m$ trials. Further when p is defined as the limit of s/n as n tends to infinity, it naturally means that s should lie between $n(p-\epsilon)$ and $n(p+\epsilon)$. It is only this principle that is revealed even in the case of the m trials succeeding the first n trials; and shows that μ should lie between $m(p-\epsilon)$ and $m(p+\epsilon)$ when m is large. There is nothing wrong in this. The fact that the number of successes μ may be anywhere from 0 to m in m trials, is equally true in the case of the first n trials and s can take any value from 0 to n. This can never be the case. In fact the statistical definition of probability shows that s will not take any random value from 0 to n but will lie between $n(p-\epsilon)$ and $n(p+\epsilon)$; or even μ will lie between $m(p-\epsilon)$ and $m(p+\epsilon)$ when m is large. Whether it is the first n trials or last m trials, the same principle holds; the quasi-limit definition introduced in the above paper is only an unwanted burden on a pure, simple yet precise definition. On the other hand if it is conceded that μ can vary from 0 to m (*i.e.*, s can take any value from 0 to n) and recognise the possible randomness of the number of successes, we can immediately close all our books on actuarial science, stat-