

LETTERS TO THE EDITOR

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A GENERALISATION OF LAPLACE'S TRANSFORM

1. Transforms of the type

$$\phi(p) = p \int_0^{\infty} F(xp) f(x) dx$$

have long been known and the special case of this when $F(xp) \equiv e^{-xp}$ and hence

$$\phi(p) = p \int_0^{\infty} e^{-xp} f(x) dx \tag{1}$$

has led to the subject of Operational Calculus, a powerful tool for tackling problems in Mathematical Physics and on which exists a good deal of literature. The relation (1) is known as Laplace's integral. It is possible to generalise¹ Laplacian Transform and the object of this note is to give an interesting generalisation and to introduce a new calculus based on this generalised Laplace's Transform.

2. If we take

$$F(xp) \equiv (2xp)^{-\frac{1}{2}} W_{k,m}(2xp)$$

where² $W_{k,m}(x)$ denotes a Whittaker Function, we obtain the transform

$$\phi_m^k(p) = p \int_0^{\infty} (2xp)^{-\frac{1}{2}} W_{k,m}(2xp) f(x) dx \tag{2}$$

For $k = \frac{1}{2}$ and $m = \pm \frac{1}{2}$ when $(2xp)^{-\frac{1}{2}} W_{k,m}(2xp)$ degenerates into e^{-xp} we get Laplace's Transform.

We shall call $\phi_m^k(p)$ as the Whittaker or $W_{k,m}$ -transform of $f(x)$ and $f(x)$ the original of $\phi_m^k(p)$ in this new transform.

In view of the involved nature of the function $(2xp)^{-\frac{1}{2}} W_{k,m}(2xp)$ as compared to the exponential function e^{-xp} , theorems based on this generalised Laplace's transform are not so easy to prove. We give, for the present, without proof, five theorems for this new Calculus.

Theorem 1A. If $\phi_m^k(p)$ is the Whittaker Transform of $f(x)$, then $\left(-p \frac{d}{dp}\right)^n \phi_m^k(p)$ is the Whittaker Transform of $\left(x \frac{d}{dx}\right)^n f(x)$.

Theorem 2A. If $\phi_m^k(p)$ and $\phi_{m'}^{k'}(p)$ are the $W_{k,m}$ - and $W_{k',m'}$ -transforms of $f(x)$ and $\psi(x)$ respectively, then

$$\int_0^{\infty} \phi_m^k(x) \psi(x) \frac{dx}{x} = \int_0^{\infty} \phi_{m'}^{k'}(x) f(x) \frac{dx}{x}$$

This theorem may be considered as a Parseval Theorem in the Theory of Whittaker Transforms.

Theorem 1B. If $\phi_m^k(p)$ is the Whittaker transform of $f(x)$, then

$$\int_0^{\infty} \phi_m^k(x) \frac{dx}{x} = \frac{\Gamma\left(\frac{1}{2} - m\right) \Gamma\left(\frac{1}{2} + m\right)}{2 \Gamma\left(\frac{1}{2} - k\right)}$$