LETTERS TO THE EDITOR

A PECULIAR GAP-FILLING PROCESS FOR POWERS OF $(9)_n$

IN the *B.U.J.*,* Mr. Kaprekar has considered a few examples in the gap-filling process for powers of $(9)_n$. In that note, he has taken powers only upto 5. In this note, I establish a method for the gap-filling process whatever be the power. Moreover in this method, we need not calculate the value of the different powers of 9, which are necessary in Mr. Kaprekar's method.

Notations.—(1) $(9)_n$ stands for the digit 9 repeated n times and $(9)^k_n$ stands for the k^{th} power of $(9)_n$.

(2) p is defined by the relation,

$$10^{p-1} < {}^kC_r \leqslant 10^p, \text{ where } r = \frac{k}{2} \text{ if } k \text{ is even,}$$

$$\text{and } = \frac{k-1}{2} \text{ if } k \text{ is odd.}$$

(3) k and n are positive integers and $n \geqslant p$.

(4) When we call (x) a p digit number; if x does not contain p digits. we mean that the necessary number of zeros are prefixed to x so as to make it a p digit number. *e.g.*, if $p = 3$ and $x = 35$, we take $(x) = (035)$.

With these notations we will show that $(9)_n{}^k = \ldots\ldots(10^p - {}^kC_1)\ldots\ldots({}^kC_2 - 1)\ldots\ldots(10^p - {}^kC_3)\ldots\ldots({}^kC_4 - 1)\ldots\ldots$ where $(10^p - {}^kC_1)$, $({}^kC_2 - 1)$ etc., are all p digit numbers and the gaps from left to right are filled in by $(9)_{n-p}$ and $(0)_{n-p}$ alternately, thus making it

$$= (9)_{n-p}(10^p - {}^kC_1)(0)_{n-p}({}^kC_2 - 1)(9)_{n-p}(10^p - {}^kC_3)(0)_{n-p}({}^kC_4 - 1).$$

The last p digits being by ${}^kC_k$ or $(10^p - {}^kC_k)$ according as k is even or odd.

Proof.—$(9)_n{}^k = (10^n - 1)^k$

$$= 10^{nk} - {}^kC_1 \cdot 10^{n(k-1)} + {}^kC_2 \cdot 10^{n(k-2)} - {}^kC_3 \cdot 10^{n(k-3)} + \ldots$$

$$= [10^n - {}^kC_1] \cdot 10^{n(k-1)} + [{}^kC_2\, 10^n - {}^kC_3] \cdot 10^{n(k-3)} + \ldots$$

$$= [(9)_{n-p}\, 10^p + (10^p - {}^kC_1)]\, 10^{n(k-1)} + [({}^kC_2 - 1) \cdot 10^n + (9)_{n-p}\, 10^p + (10^p - {}^kC_3)] \cdot 10^{n(k-3)} \ldots$$

since $10^n - 10^p = (9)_{n-p} \cdot 10^p$

$$= [(9)_{n-p} \cdot (10^p - {}^kC_1)]\, 10^{n(k-1)} + [({}^kC_{2-1})\, (9)_{n-p}\, (10^p - {}^kC_3)]\, 10^{n(k-3)} + \ldots$$

since $(10^p - {}^kC_1)$ etc are p digit numbers.

$$= (9)_{n-p}\, (10^p - {}^kC_1) \cdot (0)_{n-p}\, ({}^kC_2 - 1)\, (9)_{n-p}\, (10^p - {}^kC_3) \ldots$$

$$= \ldots (10^p - {}^kC_1) \ldots ({}^kC_2 - 1) \ldots (10^p - {}^kC_3) \ldots ({}^kC_4 - 1) \ldots,$$

where $(10^p - {}^kC_1)$, $({}^kC_2 - 1)$, etc., are p digit numbers; and the gaps are filled in with $(9)_{n-p}$ and $(0)_{n-p}$ alternately.

When $k \leqslant 5$. $p = 1$ and so the problem is simple.

Ex.—$(9)_4{}^5 = -5-9-0-4-9$ to be filled in by $(9)_3$ and $(0)_3$ alternately $= 999\,5\,000\,9\,999\,0\,000\,4\,9999$.

We find that

$p=1$ if $k \leqslant 5$ $\quad$ $p=2$ if $6 \leqslant k \leqslant 8$
$p=3$ if $9 \leqslant k \leqslant 12$ $\quad$ $p=4$ if $13 \leqslant k \leqslant 15$
$p=5$ if $16 \leqslant k \leqslant 19$ $\quad$ $p=6$ if $20 \leqslant k \leqslant 22$.

Note.—Whatever be the value of n $(> p)$ to write down $(9)_n{}^k$, we have to calculate ${}^kC_1$, ${}^kC_2$ $\ldots{}^kC_k$ only.

Trivandrum, *August* 29, 1945. S. PARAMESWARAN.

* *Bombay University Journal*, March 1945.

THE EFFECT OF COLOUR ON THE VISUAL OBSERVATION OF LONG-PERIOD VARIABLE STARS

THE part played by colour in the errors involved in the visual observation of long-period variables was pointed out by Ford.[1] In order to verify the linear relationship between colour and mean deviation as derived by him, a study of twenty stars of varying colour was made utilising the same methods of analysis. The observational data were taken from the A.A.V.S.O. Reports in *Harvard Annals*, Vol. 107, Nos. 7 and 8, and Vol. 110, Nos. 1, 5, 6, 7 and 8. The deviations for each individual