LETTERS TO THE EDITOR

INTERACTION FORMULÆ IN ANALYSIS OF VARIANCE

1. THE methods adopted by J. O. Irwin (*J.R.S.S.*, Vol. 94, 1931) in proving some of the main formulæ in Analysis of Variance indicate that he has missed the proper method of approach to these formulæ. We propose to point out here an extremely simple method of deriving them from a repeated application of the elementary result

$$S(x^2) - n(\bar{x}^2) = S(x - \bar{x})^2,$$

which we rewrite

$$S(x^2) - S(\bar{x}^2) = S(x - \bar{x})^2. \qquad (1)$$

In Irwin's notation for a set of three-way variates x_{uvw} which may be grouped according to u's, v's, w's, (uv)'s, (vw)'s and (uw)'s, we get, by applying (1) to the various groups into which the aggregate of x's may be subdivided, the following results:—

$$S(x_{uvw} - \bar{x})^2 - S(\bar{x}_{u..} - \bar{x})^2 = S(x_{uvw} - \bar{x}_{u..})^2 \qquad (2)$$

$$S(x_{uvw} - \bar{x}_{u..})^2 - S(\bar{x}_{.v.} - \bar{x})^2 = S(x_{uvw} - \bar{x}_{u..} - \bar{x}_{.v.} + \bar{x})^2 \qquad (3)$$

$$S(x_{uvw} - \bar{x}_{u..} - \bar{x}_{.v.} + \bar{x})^2 - S(\bar{x}_{..w} - \bar{x})^2 = S(x_{uvw} - \bar{x}_{u..} - \bar{x}_{.v.} - \bar{x}_{..w} + 2\bar{x})^2 \qquad (4)$$

$$S(x_{uvw} - \bar{x}_{u..} - \bar{x}_{.v.} - \bar{x}_{..w} + 2\bar{x})^2 - S(\bar{x}_{uv.} - \bar{x}_{u..} - \bar{x}_{.v.} + \bar{x})^2 = S(x_{uvw} - \bar{x}_{..w} - \bar{x}_{uv.} + \bar{x})^2 \qquad (5)$$

$$S(x_{uvw} - \bar{x}_{..w} - \bar{x}_{uv.} + \bar{x})^2 - S(\bar{x}_{.vw} - \bar{x}_{..w} - \bar{x}_{.v.} + \bar{x})^2 = S(x_{uvw} - \bar{x}_{.vw} + \bar{x}_{.v.} - \bar{x}_{uv.})^2 \qquad (6)$$

$$S(x_{uvw} - \bar{x}_{.vw} + \bar{x}_{.v.} - \bar{x}_{uv.})^2 - S(\bar{x}_{u.w} - \bar{x}_{..w} + \bar{x} - \bar{x}_{u..})^2 = S(x_{uvw} + \bar{x}_{u..} + \bar{x}_{.v.} + \bar{x}_{..w} - \bar{x}_{.vw} - \bar{x}_{u.w} - \bar{x}_{uv.} - \bar{x})^2 \qquad (7)$$

which last is the sum of squares for second-order interaction. The summation extends over all the variates x_{uvw}. It is assumed that each r-way sub-group of the same kind contains the same number of elements, $(r = 1, 2)$.

2. If the suffixes u, v, w refer respectively to rows, columns and treatments of a Latin Square, the formula for interaction in a Latin Square design is obtained by adding (2), (3) and (4) above, in the form

$$S(x_{uvw} - \bar{x})^2 - S(\bar{x}_{u..} - \bar{x})^2 - S(\bar{x}_{.v.} - \bar{x})^2 - S(\bar{x}_{..w} - \bar{x})^2 = S(x_{uvw} - \bar{x}_{u..} - \bar{x}_{.v.} - \bar{x}_{..w} + 2\bar{x})^2 \equiv D, \text{ (say)} \qquad (8)$$

If x_{ijk} be any particular variate, then

$$\frac{1}{2}\frac{\partial D}{\partial x_{ijk}} = S\left[(x_{uvw} - \bar{x}_{u..} - \bar{x}_{.v.} - \bar{x}_{..w} + 2\bar{x})\left(\frac{\partial x_{uvw}}{\partial x_{ijk}} - \frac{\partial \bar{x}_{u..}}{\partial x_{ijk}} - \frac{\partial \bar{x}_{.v.}}{\partial x_{ijk}} - \frac{\partial \bar{x}_{..w}}{\partial x_{ijk}} + 2\frac{\partial \bar{x}}{\partial x_{ijk}}\right)\right]$$

$$= S(x_{ijk} - \bar{x}_{i..} - \bar{x}_{.j.} - \bar{x}_{..k} + 2\bar{x});$$

since the expression

$$(x_{ijk} - \bar{x}_{i..} - \bar{x}_{.j.} - \bar{x}_{..k} + 2\bar{x})$$

vanishes when summed over either the i-th row or the j-th column or the k-th treatment.

Maharaja's College,
Mysore, A. A. KRISHNASWAMI AYYANGAR.
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SATURATION IN THE *LIGHT-EFFECT* UNDER ELECTRIC DISCHARGE

THAT the magnitude of this phenomenon, *viz.*, a current decrease Δi due to irradiation, increases at first rapidly and then slowly as the