LETTERS TO THE EDITOR

INTERACTION FORMULÆ IN ANALYSIS OF VARIANCE

1. THE methods adopted by J. O. Irwin (*J.R.S.S.*, Vol. 94, 1931) in proving some of the main formulæ in Analysis of Variance indicate that he has missed the proper method of approach to these formulæ. We propose to point out here an extremely simple method of deriving them from a repeated application of the elementary result

$$S(x^2) = n(\bar{x}^2) + S(x - \bar{x})^2,$$

which we rewrite

$$S(x^2) = S(\bar{x}^2) + S(x - \bar{x})^2. \qquad (1)$$

In Irwin's notation for a set of three-way variates x_{uvw} which may be grouped according to u's, v's, w's (uv)'s, (vw)'s and (uw)'s, we get, by applying (1) to the various groups into which the aggregate of x's may be subdivided, the following results:—

$$S(x_{uvw} - \bar{x})^2 = S(x_{uv\cdot} - \bar{x})^2 + S(x_{uvw} - \bar{x}_{uv\cdot})^2 \qquad (2)$$

$$S(x_{uvw} - \bar{x}_{uv\cdot})^2 = S(\bar{x}_{\cdot v\cdot} - \bar{x})^2 + S(x_{uvw} - \bar{x}_{u\cdot\cdot} - \bar{x}_{\cdot v\cdot} + \bar{x})^2 \qquad (3)$$

$$S(x_{uvw} - \bar{x}_{u\cdot\cdot} - \bar{x}_{\cdot v\cdot} + \bar{x})^2 = S(x_{uv\cdot} - \bar{x}_{u\cdot\cdot} - \bar{x}_{\cdot v\cdot} + \bar{x})^2 + S(x_{uvw} - \bar{x}_{uv\cdot} - \bar{x}_{\cdot\cdot w} + \bar{x})^2 \qquad (4)$$

$$S(x_{uvw} - \bar{x}_{u\cdot\cdot} - \bar{x}_{\cdot v\cdot} - \bar{x}_{\cdot\cdot w} + 2\bar{x})^2 = S(x_{uv\cdot} - \bar{x}_{u\cdot\cdot} - \bar{x}_{\cdot v\cdot} + \bar{x})^2 + S(x_{uvw} - \bar{x}_{uv\cdot} - \bar{x}_{\cdot\cdot w} + \bar{x})^2 \qquad (5)$$

$$S(x_{uvw} - \bar{x}_{\cdot v\cdot} - \bar{x}_{\cdot\cdot w} + \bar{x})^2 = S(\bar{x}_{\cdot v\cdot} - \bar{x}_{\cdot\cdot\cdot} - \bar{x}_{\cdot v\cdot} + \bar{x})^2 + S(x_{uvw} - \bar{x}_{\cdot v\cdot} - \bar{x}_{u\cdot\cdot} + \bar{x}_{\cdot\cdot\cdot})^2 \qquad (6)$$

$$S(x_{uvw} - \bar{x}_{\cdot v\cdot} - \bar{x}_{\cdot\cdot w} + \bar{x})^2 = S(\bar{x}_{u\cdot w} - \bar{x}_{u\cdot\cdot} - \bar{x}_{\cdot\cdot w} + \bar{x})^2 + S(x_{uvw} - \bar{x}_{u\cdot w} - \bar{x}_{\cdot v\cdot} + \bar{x}_{\cdot\cdot\cdot})^2 \qquad (7)$$

which last is the sum of squares for second-order interaction. The summation extends over all the variates x_{uvw}. It is assumed that each r-way sub-group of the same kind contains the same number of elements, $(r = 1, 2)$.

2. If the suffixes u, v, w refer respectively to rows, columns and treatments of a Latin Square, the formula for interaction in a Latin Square design is obtained by adding (2), (3) and (4) aboye, in the form

$$S(x_{uvw} - \bar{x})^2 - S(\bar{x}_{u\cdot\cdot} - \bar{x})^2 - S(\bar{x}_{\cdot v\cdot} - \bar{x})^2 - S(\bar{x}_{\cdot\cdot w} - \bar{x})^2 = S(x_{uvw} - \bar{x}_{u\cdot\cdot} - \bar{x}_{\cdot v\cdot} - \bar{x}_{\cdot\cdot w} + 2\bar{x})^2 \equiv D, \text{ (say)} \qquad (8)$$

If x_{ijk} be any particular variate, then

$$\frac{1}{2}\frac{\partial D}{\partial x_{ijk}} = S\left[(x_{uvw} - \bar{x}_{u\cdot\cdot} - \bar{x}_{\cdot v\cdot} - \bar{x}_{\cdot\cdot w} + 2\bar{x})\left(\frac{\partial x_{uvw}}{\partial x_{ijk}} - \frac{\partial \bar{x}_{u\cdot\cdot}}{\partial x_{ijk}} - \frac{\partial \bar{x}_{\cdot v\cdot}}{\partial x_{ijk}} - \frac{\partial \bar{x}_{\cdot\cdot w}}{\partial x_{ijk}} + 2\frac{\partial \bar{x}}{\partial x_{ijk}}\right)\right]$$
$$= S(x_{ijk} - \bar{x}_{i\cdot\cdot} - \bar{x}_{\cdot j\cdot} - \bar{x}_{\cdot\cdot k} + 2\bar{x});$$

since the expression

$$(x_{ijk} - \bar{x}_{i\cdot\cdot} - \bar{x}_{\cdot j\cdot} - \bar{x}_{\cdot\cdot k} + 2\bar{x})$$

vanishes when summed over either the i-th row or the j-th column or the k-th treatment.

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SATURATION IN THE *LIGHT-EFFECT* UNDER ELECTRIC DISCHARGE

THAT the magnitude of this phenomenon, *viz.*, a current decrease Δi due to irradiation, increases at first rapidly and then slowly as the

light-intensity I is increased progressively, was observed earlier;[1,2,3,4,5,8,9] as also its pronounced occurrence in a chlorine-filled discharge tube of the *full-* or the *semi*-ozonizer type owing to the large surface exposed. Subsequent results were interesting for Δi due to more than one light-source used simultaneously.

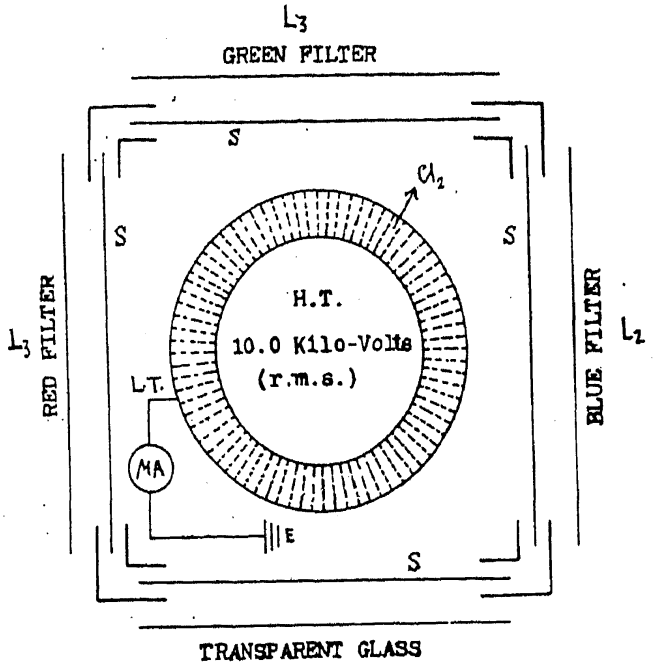

FIG. I

The glass ozonizer, kept at the axis of a rectangular box with movable side-shutters (S in Fig. 1) could be irradiated by one or more of 200-watt incandescent bulbs L_1, L_2, L_3, L_4. At 10 kilo-volts (r.m.s.), Δi corresponded to 29 per cent. with L_1 alone; with $L_1 + L_2$ used simultaneously, Δi increased to 32 per cent.; it was 35 per cent. and 36 per cent. due to the combined irradiation from three and four bulbs respectively. Thus, for about a fourfold increase of I, the light-effect rises from 29 to 36, that is, by 24 per cent.; corresponding to the final 100 per cent. rise in I, that in Δi is only 3 per cent. These results reveal an approach to saturation or some limiting condition. They cannot be ascribed to any 'fatigue' effects familiar in photo-electric phenomena, especially since restoration of the discharge current to its original value in dark after shutting off the light-source at any of the above stages of intensity-variation, was complete and sensibly instantaneous.[1,2,3]

Light filters (glass) fixed as shown in Fig. 1 enabled observations of Δi due to exposure in one or more parts of the spectrum. The light-effect under the unfiltered, *i.e.*, the white light (3700-7800 Å) using L_1 was 29 per cent.; in the filtered blue (4120-4960 Å), green (5100-5780 Å) and red (6000-7300 Å), Δi was 28 per cent., 6 per cent. and 3 per cent. respectively. The light-effect due to $L_1 + L_2$ used simultaneously almost equalled that due to the direct white (*i.e.*, unfiltered) from L_1 combined with the filtered blue from L_2. This is in agreement with the earlier results that the light-effect of the blue component is almost equal to that under the unfiltered white especially at large I due, in part, to saturation. Furthermore, Δi produced under blue (or white) showed no appreciable increase when irradiated in addition, in the green or/and in the red. Similarly, Δi due to green (using L_3 and the filter) was but slightly increased, when irradiated simultaneously with the red (L_4 and the filter). It is generalised that, presumably due to saturation, the light-effect by simultaneous irradiations in different spectral regions is less than the sum of the corresponding Δi produced separately. It is remarkable that this deduction holds for such widely separated regions of the spectrum as the visible and the X-rays.[5]

The above facts are in striking contrast with those characteristic of the classical photo-electric effect, not only in the sense that it entails a conductivity rise but also because the magnitude of the former is affected markedly owing to a simultaneous irradiation in the long-wave region. Thus, for example, in certain cells containing composite surfaces, exposure to the infra-red *decreases* the photo-electric effect by 50 per cent.[6] In numerous cases of photo-conductivity (the volume or internal photo-electric effect), however, a simultaneous exposure in the red or infra-red is known to increase the conductivity by over 100 per cent. This is attributed to a motion towards the cathode of the positive ions left as 'space charge' during the primary irradiation of the crystal. Such a mechanism cannot hold appreciably for the present phenomenon, which is produced predominantly in a gas under electric discharge. The above type of influence on the gas-solid interface presented by the container walls is, however, not unlikely, since the nature of this interface and its immediate neighbourhood are an important determinant of Δi. It may also be emphasised that in notable contrast with especially the surface photo-electric effect, *in vacuo*, the non-linearity of the Δi–I relationship—almost invariably the curve is concave to the I-*axis*—established over a wide range of conditions studied in these Laboratories, can now be taken as a general characteristic of the present phenomenon. This together with the finding that Δi is markedly sensitive to pressure changes of the excited gas[1,2,4] suggests, that secondary effects perhaps of the collision type are superimposed on a primary *quantaic* change in the occurrence of this phenomenon.[1,2]

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1. Joshi, *Presi. Add., Chem. Sec., Indian Sci. Cong.*, 1943. 2. — *B.H.U. Journ.*, 1943, **8**, 199. 3. —, *Curr. Sci.*, 1944, **13**, 253. 4. — and Deo, *Nature*, 1943, **151**, 561. 5. —, *Curr. Sci.*, 1944, **13**, 273. 6. Olpin, *Phys. Rev.*, 1930, **36**, 376. 7. Gudden and Pohl, *Zeits. f. Phys.*, 1924 **20**, 14. 8. Deo, *Curr. Sci.*, 1944, **13**, 44. 9. Joshi and Deo, *Nature*, 1944, **153**, 434.