THE DISTRIBUTION OF THE MEAN OF SAMPLES FROM A RECTANGULAR POPULATION

IRWIN[1] has obtained the distribution of the mean of samples from a rectangular population by using characteristic functions. It has been shown in this note that this distribution can be obtained by using either of the two sets of multiple integrals given below :—

$$\frac{d}{d\bar{x}}\int_0^{n\bar{x}}\int_0^{n\bar{x}-x_1}\cdots\int_0^{n\bar{x}-x_1-x_2\ldots x_{n-1}} dx_1\,dx_2\ldots dx_n = \frac{n^n}{\Gamma_n}\,\bar{x}^{n-1} \qquad (1)$$

$$\frac{d}{d\bar{x}}\int_1^{n\bar{x}}\int_0^{n\bar{x}_1-x_1}\cdots\int_0^{n\bar{x}-x_1-x_2\ldots x_{n-1}} dx_1\,dx_2\ldots dx_n = \frac{n^n}{\Gamma_n}\left(\bar{x}-\frac{1}{n}\right)^{n-1} \qquad (2)$$

$$\frac{d}{d\bar{x}}\int_1^{n\bar{x}}\int_1^{n\bar{x}-x_1}\cdots\int_0^{n\bar{x}-x_1-x_2\ldots x_{n-1}} dx_1\,dx_2\ldots dx_n = \frac{n^n}{\Gamma_n}\left(\bar{x}-\frac{2}{n}\right)^{n-1} \qquad (3)$$

....

.... (A)

Same as (1) of (A) (1)

$$\frac{d}{d\bar{x}}\int_0^{1}\int_0^{n\bar{x}-x_1}\cdots\int_0^{n\bar{x}-x_1-x_2\ldots x_{n-1}} dx_1\,dx_2\ldots dx_n = \frac{n^n}{\Gamma_n}\left[\bar{x}^{n-1}-\left(\bar{x}-\frac{1}{n}\right)^{n-1}\right] \qquad (2)$$

$$\frac{d}{d\bar{x}}\int_0^{1}\int_0^{1}\cdots\int_0^{n\bar{x}-x_1-x_2\ldots x_{n-}} dx_1\,dx_2\ldots dx_n = \frac{n^n}{\Gamma_n}\left[\bar{x}^{n-1}-2\,c_1\left(\bar{x}-\frac{1}{n}\right)^{n-1}+\left(\bar{x}-\frac{2}{n}\right)^{n-1}\right] \qquad (3)$$

$$\frac{d}{d\bar{x}}\int_0^{1}\int_0^{1}\int_0^{1}\cdots\int_0^{n\bar{x}-x_1-x_2\ldots x_{n-1}} dx_1\,dx_2\ldots dx_n = \frac{n^n}{\Gamma_n}\left[\bar{x}^{n-1}-3\,c_1\left(\bar{x}-\frac{1}{n}\right)^{n-1}+3\,c_2\left(\bar{x}-\frac{2}{n}\right)^{n-1}-\left(\bar{x}-\frac{3}{n}\right)^{n-1}\right] \qquad (4)$$

....

$$\frac{d}{d\bar{x}}\int_0^{1}\int_0^{1}\cdot\cdot\text{p times}\cdot\cdot\int_0^{n\bar{x}-x_1-x_2\ \ x_{n-1}} dx_1\,dx_2\cdot\cdot dx_n = \frac{n^n}{\Gamma_n}\left[\bar{x}^{n-1}-p\,c_1\left(\bar{x}-\frac{1}{n}\right)^{n-1}+p\,c_2\left(\bar{x}-\frac{2}{n}\right)^{n-1}-p\,c_3\left(\bar{x}-_n\right)^{n-1}\cdots(-)^p\left(x-\frac{p}{n}\right)^{n-1}\right]\cdot\cdot\ (p+1) \qquad (B)$$

The probability of a sample of size n, with values lying between x_1, x_1+dx_1; x_2, x_2+dx_2; $\cdot\cdot\ \cdot\cdot x_n, x_n+dx_n$ is

$$dx_1\,dx_2\ldots\ldots dx_n$$

Using (1) of A, the distribution of

$$\bar{x}=\frac{x_1+x_2\ldots x_n}{n} \text{ is}$$

$$\frac{n^n}{\Gamma_n}\,\bar{x}^{n-1}\,dx.$$

But it can be seen that this distribution is valid only so long as $nx \leqslant 1$. When $n\bar{x} > 1$ some of the observed values $x_1, x_2, \ldots x_n$ can be > 1 and they are also included in the distribution obtained. Hence the distribution of the mean for the interval 0 to $\frac{1}{n}$ is

$$\frac{n^n}{\Gamma_n}\,\bar{x}^{n-1}\,d\bar{x} \qquad (C)$$

When $\bar{x}$ lies between $\frac{1}{n}$ and $\frac{2}{n}$, $n\bar{x}$ lies between 1 and 2 and it is possible that one of the n observed values is > 1 and the distribution of $\bar{x}$ corresponding to these values should be subtracted from (C). It can be seen the distribution of $\bar{x}$ when one of the observations is > 1 is given by (2) of (A). As there are ${}_nC_1$ ways of having one of the observations > 1, the distribution of $\bar{x}$ for the interval $\frac{1}{n}$ to $\frac{2}{n}$ is

$$\frac{n^n}{\Gamma_n}\left[\bar{x}^{n-1}-{}_nC_1\left(\bar{x}-\frac{1}{n}\right)^{n-1}\right]dx \qquad (D)$$

It may be noted that (D) is also equal to ${}_nC_1\cdot(2)-{}_nC_0\cdot{}_{n-1}C_1\cdot(1)$, where (2) and (1) refer to (B).

When $\bar{x}$ lies between $\frac{2}{n}$ and $\frac{2}{n}$, two of the n observed values can be > 1 and the distribution can be got by subtracting the distribution corresponding to this portion from (D). First we note that when two of the observations are > 1, the distribution is given by (3) of (A). Out of the n observations, the number of combinations having two observations > 1 included in (1) and (2) of (A) is ${}_nC_2$ and $n\ (n-1)$ respectively. Adding them with the proper signs we find that $-\frac{n\ (n-1)}{2}$ sets of observations having two values > 1 are to be subtracted from (D) to have the distribution of $\bar{x}$ in the range $\frac{2}{n}$ to $\frac{3}{n}$.

Hence the distribution of $\bar{x}$ in the interval $\frac{2}{n}$ to $\frac{3}{n}$ is

$$\frac{n^n}{\Gamma_n}\left[\bar{x}^{n-1}-{}_nC_1\left(\bar{x}-\frac{1}{n}\right)^{n-1}+ +{}_nC_2\left(\bar{x}-\frac{2}{n}\right)^{n-1}\right]d\bar{x}. \qquad (E)$$

It will be seen that (E) is also equal to ${}_nC_2\ (3)-{}_nC_1\cdot{}_{n-2}C_1\cdot(2)+{}_nC_0\cdot{}_{n-1}C_2\ (1)$, where (3), (2) and (1) refer to (B).

When $\bar{x}$ lies between $\frac{3}{n}$ and $\frac{4}{n}$ three of the observed values can be > 1, and the distribution corresponding to these values is to be subtracted from (E) to get the correct distribution for the interval $\frac{3}{n}$ to $\frac{4}{n}$ of $\bar{x}$. As in the previous cases, when three of the observations are > 1, the distribution for such cases is given by (4) of (A). The number of combinations having three observations > 1 included in (1), (2) and (3) of A is

$${}_nC_3,\ \frac{n(n-1)(n-2)}{|1\,|2}\ \text{and}\ \frac{n(n-1)(n-2)}{|2}$$

respectively. Adding them with the proper signs it will be seen that ${}_nC_3$ samples with three observations in each > 1 are to be subtracted from (E) and the final distribution of x for the interval $\frac{3}{n}$ to $\frac{4}{n}$ becomes

$$\frac{n^n}{\Gamma_n}\left[x^{n-1} - {}_nC_1\left(x-\frac{1}{n}\right)^{n-1} + {}_nC_2\left(x-\frac{2}{n}\right)^{n-1} - {}_nC_3\left(x-\frac{3}{n}\right)^{n-1}\right]dx \qquad \text{(F)}$$

As in the previous cases (F) can be shown to be equal to

$${}_nC_3\cdot(4) - {}_nC_2\cdot{}_{n-3}C_1\cdot(3) + {}_nC_1\cdot{}_{n-2}C_2\cdot(2) - {}_nC_0\cdot{}_{n-1}C_3\cdot(1),$$

where (4), (3), (2) and (1) refer to (B).

The above argument can be extended to the case of x lying in the interval $\frac{p}{n}$ and $\frac{p+1}{n}$, p being < n. The number of combinations having p observations > 1 to be subtracted from the distribution for the interval $\frac{p-1}{n}$ and $\frac{p}{n}$ is

$${}_nC_p - {}_nC_1\cdot{}_{n-1}C_{p-1} + {}_nC_2\cdot{}_{n-2}C_{p-2} \cdots + (-)^{p-1}{}_nC_{p-1}\cdot(n-p).$$

This is equal to $+$ or $-{}_nC_p$ according as p is odd or even. Hence the distribution for the interval $\frac{p}{n}$ and $\frac{p+1}{n}$ is obtained by subtracting $(-)^{p-1}{}_nC_p$ times the distribution of samples having p observations > 1 from the distribution for the preceding interval.

In general the distribution of x for the interval $\frac{p}{n}$ to $\frac{p+1}{n}$ is also given by

$${}_nC_p\cdot(p+1) - {}_nC_{p-1}\cdot{}_{n-p}C_1\cdot(p) + {}_nC_{p-2}\cdot{}_{n-p+1}C_2\cdot(p-1) \ldots + (-)^p\,{}_nC_0\cdot{}_{n-1}C_p\cdot(1),$$

where $(p+1)$, (p), ... (1) refer to the integrals given in (B).

The above method of approach will lead to the distribution of the means of Type II and I curves which will be useful in getting the distribution of the mean of $Y = \left(1 + \frac{T^2}{N-1}\right)$, where T^2 refers to Hotelling's[2] T^2. As the actual expressions for the distribution of $\bar{y}$ are complicated they will be dealt with in another communication.

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1. Irwin, J. O., *Biometrika*, 1927, **19**, 225. 2. Hotelling, H., *Annals of Math. Sta.*, 1931, **2**, 360.

NEW BANDS OF THE HgBr MOLECULE

IN the course of a reinvestigation of the band systems of the HgBr molecule, in order to bring them into conformity with those of HgCl, the authors have obtained a new band system in the region λ 2470–λ 2430, consisting of diffuse and headless bands. About thirty bands could be measured and assigned to the three sequences (0,1), (0,0) and (1,0). The intensity distribution gives a narrow Condon Parabola. The following vibrational constants have been determined:—

$$\nu_e = 40720 \qquad \omega_e' = 166\cdot0 \qquad \omega_e'' = 183\cdot0$$
$$x_e'\omega_e' = 1\cdot1 \qquad x_e''\omega_e'' = 2\cdot0$$

The final state of this system appears to be the same as the final state of the class I system of the HgBr bands, studied previously by Wieland[1] and by Sastry.[2] Full details will be published shortly.

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1. Wieland, *Helv. Acta. Phys.*, 1929, **2**, 46 and 77.
2. Sastry, *Nat. Inst. Sci. Ind. Proc.*, 1941, **7**, 353.

MAGNETISM OF GERMANIUM

FROM several points of view, germanium is an interesting element. It crystallizes in the diamond structure. But in electrical properties, it behaves like a semiconductor. Unlike diamond, germanium is soft and has a metallic lustre. One would expect these properties to be reflected in the magnetic behaviour of germanium.

This element has been studied by Owen (1912) who obtained − 0·12* as its specific susceptibility. A redetermination of the susceptibility seemed desirable since a sample of Hilger's spectroscopic brand was available. Curie's method was adopted, using water as the standard. The element showed a trace of ferromagnetic impurity, for which due correction was made. The average specific susceptibility obtained with two pieces was found to be − 0·147. This gives for the atomic susceptibility the value − 10·7.

Diamond, silicon, germanium and grey tin belong to the same class of elements from the point of view of crystal chemistry. It is obvious, however, that germanium stands unique on account of some of its properties being absent in the other cases.

A calculation of the atomic susceptibility of germanium on the basis of Slater's method, as modified by Angus (1932) gives − 40·9. On the same basis, the ionic susceptibility of Ge^{++} (which is in $^1S_0$ state) works to − 16·8 while the experimental value for the metal is − 10·7. These results suggest that the metal contains Ge^{++} ions, with the two $4p$ electrons showing evidence of both valence characteristics and metallic bond. That the valence characteristics are predominant and the metallic bond subdued are apparent from the following facts.

Owen (1912) found that when the element is melted, there is a large rise in the diamagnetic susceptibility. In the case of covalent or homopolar combination, the increase of diamagnetism is accompanied by a link depression. On melting, this depression should partly vanish and the valence electrons would evince a larger diamagnetism.