LETTERS TO THE EDITOR

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A NOTE ON THE GENERALIZED VARIANCE OF MULTIVARIATE POPULATIONS

In the case of multivariate populations the generalized variance for all practical purposes takes the place of the variance of single variate populations. This is obvious when we consider Fisher's t^2 and Hotelling's T^2. The present note gives the relationship between the generalized variance and the residual variances (on the basis of the size of the sample and not on the degrees of freedom) of the regression equations between the various variables.

The determinant

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

where $a_{ij} = \frac{1}{N} \Sigma (x_{ir} - \bar{x}_i)(x_{jr} - \bar{x}_j)$, is called the generalized variance of a multivariate sample of size N and of n variables.

It can be shown that the above determinant is equal to

$$s_1^2 \, s_{2\cdot1}^2 \, s_{3\cdot21}^2 \cdots \cdots s_{n\cdot12\cdots n-1}^2,$$

where $s_{r\cdot12\ldots r-1}^2$ is Yule and Kendall's* notation for the residual variance of x_r expressed as a linear function of $x_1, \ldots\ldots x_{r-1}$.

The importance of this result lies in the fact that it will lead to the solution of various distribution problems connected with multivariate population without resorting to hypergeometry.

Imperial Agricultural Research Institute, New Delhi, April 19, 1944. P. V. Krishna Iyer.

* Yule, G., Udny and Kendall, M. G., *An Introduction to the Theory of Statistics*, Griffin & Co., Ltd., London, 1940.

THE PRINCIPAL MAGNETIC SUSCEPTIBILITIES OF METAL CRYSTALS

Most of the work on the determination of the principal magnetic susceptibilities of metal crystals has been carried out by preparing single crystals in the form of long rods.[1] The crystal is usually mounted with the long axis perpendicular to the magnetic lines of force of a magnetic field. The Gouy force on the specimen is studied at different orientations, when it is rotated through 360° about its long axis. From these results it is possible to calculate the principal magnetic susceptibilities, provided we know also *either* the angle ϕ between the principal crystalline axis and the long axis *or* the magnetic susceptibility of the polycrystalline specimen. The susceptibilities obtained by the Gouy method are volume susceptibilities. To determine the mass values, the density of the material has to be determined. It is well known that the actual determination of the density value causes considerable error in the final values of the principal mass susceptibilities.

Shoenberg and Uddin[2] made small beads of bismuth crystals, fixed the trigonal axis by etching and determined the principal mass susceptibilities by the Sucksmith ring balance.

The very elegant torsional method developed by Krishnan[3] was applied by John[4] to the study of the temperature variation of the magnetic anisotropy of bismuth crystal. Further study of metal crystals by Krishnan's method appears to be of great value since the principal mass susceptibilities could be directly and accurately determined. If $\chi_{||}$ and $\chi_{\perp}$ are the mass susceptibilities parallel and perpendicular to the crystalline axis, Krishnan's method enables us to determine their difference $\chi_{||} \sim \chi_{\perp}$ directly. This difference is studied at different field strengths. Curie's method may then be employed to determine the mean susceptibility χ_{mean}, also at different field strengths. Since $\chi_{mean} = 1/3\ (\chi_{||} + 2\chi_{\perp})$, it is easy to calculate $\chi_{||}$ and $\chi_{\perp}$ at different field strengths.

If the specimens contain traces of ferro-magnetic impurities, the principal mass susceptibilities may be determined, as usual, by extrapolating the χ, 1/H graphs.

Test experiments have been carried out with crystals of bismuth, zinc, cadmium and tellurium. The first three crystals have prominent cleavage in the basal plane. Small pieces from single crystal rods could be easily obtained. In the case of tellurium, the author and Govindarajan[5] had prepared tellurium crystals in the form of long rods, for magnetic measurements. One of these crystals, for which ϕ was 30°, was cut at the proper angle and the end faces of the small rods were etched. Bismuth and tellurium showed no ferromagnetic impurities while zinc and cadmium had minute traces. Very fine quartz fibres were used to suspend the crystals in the uniform magnetic field between the pole-pieces of a Pye electromagnet.

The results obtained are given below.

Crystal	Investigators	$\chi_{\parallel} \times 10^6$	$\chi_{\perp} \times 10^6$
Bismuth	Focke[6]	− 1·053	− 1·482
	John[4]	− 1·05	− 1·45
	Author	− 1·058	− 1·495
Zinc	McLennan, Ruedy and Cohen[7]	− 0·190	− 0·145
	Rao[8]	− 0·202	− 0·149
	Author	− 0·206	− 0·147
Cadmium	McLennan, Ruedy and Cohen[7]	− 0·261	− 0·160
	Rao and Sriraman[9]	− 0·223	− 0·163
	Author	− 0·234	− 0·159
Tellurium	Rao and Govindarajan[5]	− 0·329	− 0·296
	Author	− 0·342	− 0·289

A careful estimate of the errors involved brings the margin of error in the author's values to one per cent.

It is found that the results obtained by Krishnan's method agree satisfactorily with those by other methods. In the case of tellurium, the present investigation confirms the small anisotropy previously recorded by Rao and Govindarajan.[5] It is proposed to apply this method to further studies of crystal magnetism of metals, particularly from the point of view of the influence of impurity traces on magnetic anisotropy.

I thank Mr. H. S. Venkataramian for much valuable help.

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1. For details see Bates, *Modern Magnetism*, p. 144. 2. *Proc. Roy. Soc.*, 1936, **A 156**, 687. 3. *Phil. Trans. Roy. Soc.*, 1935, 234, 265. 4. *Z. Krist.*, 1939, **101**, 337 5. *Proc. Ind Acad. Sc.*, 1939, **10**, 235. 6. *Phy. Rev.*, 1930, **36**, 319. 7. *Proc. Roy. Soc.*, 1928, **121**, 9. 8. *Proc. Ind. Acad. Sc.*, 1936, **4**, 186. 9. *Proc. Roy. Soc.*, 1938, **A 166**, 325.

VAN DER WAALS' COHESION CONSTANT

It has been known for a long time that there is a close relation between surface tension and Van der Waals' constants '*a*' and '*b*'. Van der Waals himself on certain assumptions deduced that

$$\sigma_0 = A_1 \theta_c V_c^{-2/3} = Aa/b^{5/3}, \qquad (1)$$

where A_1 and A are constants, θ_c and V_c the critical temperature and critical volume respectively, and σ_0 is defined by the fundamental (empirical) relation

$$\sigma = \sigma_0 (l - \theta)^n, \qquad (2)$$

θ being the reduced temperature and n a constant which is nearly equal to 1·2 for all unassociated liquids. Ferguson[1] in a recent paper has found that the experimental values of σ_0, θ_c and V_c are in better accord with the relation

$$\sigma_0 = 3\cdot 12\ \theta_c V_c^{-0\cdot 55}. \qquad (3)$$

Recently we have been investigating the properties of liquids on the 'hole' model originally given by Fürth[2] and his collaborators in a series of papers. In this connection we were led to a simple *theoretical* relation between Van der Waals' cohesion constant and the surface tension. In a paper to be published shortly we have constructed and solved the Schrödinger equation for a 'hole' in a liquid and have determined the eigenvalues for its energy. The energy values E_n are given approximately by the expression

$$E_n = 3\cdot 61 \left(n + \frac{7}{10}\right)^{4/7} \frac{\sigma^{5/7} h^{4/7}}{\rho^{2/7}}, \qquad (4)$$

where h is the Plank constant and ρ the density of the liquid. The theory gives for the intrinsic pressure for Van der Waals' cohesion constant the expressions