LETTERS TO THE EDITOR

DISTRIBUTION OF FISHER'S g_1 FOR SAMPLES OF THREE FROM A CONTINUOUS RECTANGULAR DISTRIBUTION

An easy method to find out the distribution of Fisher's g_1 for samples of three from continuous populations appears possible from the following considerations.

Let x_1, x_2, x_3 be the sequence in which the three individuals are observed and let

$$\tau_i=\frac{x_i-\bar{x}}{s} \text{ where } \bar{x} \;\; \tfrac{1}{3}\sum_{i=1}^{3} x_i$$

$$\text{and } s^2=\tfrac{1}{3}\sum_{i=1}^{3}(x_i-\bar{x})^2. \qquad (1)$$

It is easy to see that if the τ_i's are arranged in ascending order of absolute magnitude forming the sequence τ_1', τ_2', τ_3'

$$\text{then } 0 \leqslant |\tau_1'| \leqslant \frac{1}{\sqrt{2}}\,;\; \frac{1}{\sqrt{2}} \leqslant |\tau_2'| \leqslant \sqrt{\frac{3}{2}}\,;$$

$$\sqrt{\frac{3}{2}} \leqslant |\tau_3'| \leqslant \sqrt{\ }. \qquad (2)$$

Also since

$$\sum_{i=1}^{3} \tau_i'=0\,;\; \sum_{i=1}^{3} (\tau_i')^2=3$$

$$\text{and } \sum_{i=1}^{3} (\tau_i')^3=\sqrt{\frac{3}{2}}\, g_1 \qquad (3)$$

we find that

$$g_1=\sqrt{6}\,\tau_i'\,(\tau_i'^2-\tfrac{3}{2}) \qquad (4)$$

and the distribution of g_1 can be determined once the distribution of τ_i' is known.

From the principles outlined elsewhere,[1,3] the distribution of τ_i' follows from the distribution of τ_i.

Also

$$\tau_i=\frac{\sqrt{2}\,\theta}{\sqrt{3+\theta^2}} \qquad (5)$$

where

$$\theta=\frac{x_i-\bar{x}'}{s'} \qquad (6)$$

$\bar{x}'$ and s' are respectively the mean and the standard deviation of the two observations left after excluding x_i. Hence if the probability distribution of θ is determined that of g_1 will follow immediately. This method is applied below to get the distribution of g_1 for samples of three from a continuous rectangular distribution. The notation used above has been kept up all through this note.

Rectangular Distribution

Let

$$\left.\begin{aligned} p\,(x_i) &=1 \text{ for } 0 \leqslant x_i \leqslant 1\\ &=0 \text{ elsewhere}\end{aligned}\right\} \qquad (7)$$

So

$$p\,(x_i, \bar{x}', s') \;\; 4$$

$$\left.\begin{aligned} &\text{for } 0 \leqslant x_i \leqslant 1\\ &0 \leqslant s' \leqslant \tfrac{1}{2}\,;\; s' \leqslant x' \leqslant 1-s'\end{aligned}\right\} \qquad (8)$$

Also

$$p\,(\theta, \bar{x}', s') \;\; 4\,s'$$

$$\left.\begin{aligned} &\text{for } -1\leqslant\theta\leqslant1;\; 0\leqslant s'\leqslant\tfrac{1}{2};\; s'\leqslant x'\leqslant1-s'\\ &\text{for } 1\leqslant\theta\leqslant\infty;\; 0\leqslant s'\leqslant\frac{1}{1+\theta};\\ &\qquad\qquad s'\leqslant\bar{x}'\leqslant1-\theta s'\\ &\text{for } -\infty\leqslant\theta\leqslant-1;\; 0\leqslant s'\leqslant\frac{1}{1-\theta};\\ &\qquad\qquad -\theta s'\leqslant x'\leqslant1-s'\end{aligned}\right\} \qquad (9)$$

$$\left.\begin{aligned} \therefore\;\; p\,(\theta) &=\tfrac{1}{6} \text{ for } -1\leqslant\theta\leqslant1\\ &=\tfrac{2}{3}\cdot\frac{1}{(1+\theta)^2} \text{ for } 1\leqslant\theta\leqslant\infty\\ &=\tfrac{2}{3}\cdot\frac{1}{(1-\theta)^2} \text{ for } -\infty\leqslant\theta\leqslant-1\end{aligned}\right\} \qquad (10)$$

Since

$$g_1=\frac{\sqrt{3}\,\theta\,(\theta^2-9)}{(3+\theta^2)^{3/2}} \qquad (11)$$

it follows that

$$p\,(g_1)=\frac{(3+\theta^2)^{5/2}}{27\sqrt{12}\,(1-\theta^2)} \text{ for } -1\leqslant\theta\leqslant1 \qquad (12)$$

which is equivalent to the two forms

$$\left.\begin{aligned} p\ (g_1) &= \frac{2\ (3+\theta^2)^{5/2}}{27\ \sqrt{3}\ (1+|\theta|)^2\ (\theta^2-1)} \\ &\text{for } -3 \leqslant \theta \leqslant -1;\ 1 \leqslant \theta \leqslant 3 \\ &\text{and} -\infty \leqslant \theta \leqslant -3;\ 3 \leqslant \theta \leqslant \infty. \end{aligned}\right\} \quad (13)$$

Substituting

$$\theta = \sqrt{3}\ \tan \alpha \qquad (14)$$

it is seen that these forms reduce to

$$p\ (g_1) = \frac{1}{\sqrt{12}\ \sqrt{3-g_1^2}\ \cos^2 \alpha} \quad \text{for } -\sqrt{3} \leqslant g_1 \leqslant \sqrt{3} \qquad (15)$$

where α lies between $-\frac{\pi}{6}$ and $+\frac{\pi}{6}$ and satisfies the equation

$$\sin 3\alpha = -\frac{g_1}{\sqrt{3}} \qquad (16)$$

NORMAL DISTRIBUTION

William R. Thompson (1935) has shown that

$$p\ (\tau_i') = \frac{1}{\pi \sqrt{2-\tau_i'^2}} \text{ for } -\sqrt{2} \leqslant \tau_i' \leqslant \sqrt{2} \qquad (17)$$

$$\therefore\ p\ (g_1) = \frac{1}{\pi\ \sqrt{3-g_1^2}} \text{ for } -\sqrt{3} \leqslant g_1 \leqslant \sqrt{3} \qquad (18)$$

a result established by Fisher (1930).

Comparing the distributions of g_1 for the rectangular and normal distributions it is seen that

$$p\ (g_1) \text{ rectangular} \lesseqgtr p\ (g_1) \text{ normal according as } |g_1| \lesseqgtr 1\cdot3887.$$

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November 10, 1943. C. CHANDRA SEKAR.

1. Chandra Sekar, C., and Mary Francis, G., "A Method to get the Significance Limit of a Type of Test Criteria," *Sankyha*, 1941, **5**, 165–68. 2. Fisher, R. A., "The Moments of the Distribution for Normal Samples of Measures of Departure from Normality," *Proc. Roy. Soc., London*, A, 1930, **130**, 16–28. 3. Pearson, E. S., and Chandra Sekar, C., "The Efficiency of Statistical Tests and a Criterion for the Rejection of Outlying Observations," *Biometrika*, 1936, **28**, 308–20. 4. Thompson William, R., "A Criterion for the Rejection of Observation and the Distribution of the Ratio of Deviation to Sample Standard Deviation," *Ann. Math. Stat.*, 1935, **6**, 214–19.

ON DEFINING THE α-PHONEME

FOLLOWING Professor E. W. Scripture,[1] the term 'phoneme' is applied here to 'one of a group of similar speech sounds'. It is certainly a matter of fundamental importance to be able to define precisely the positive qualities (let us call them V and C) which characterise the vowel and consonant phonemes. Linguistics has not yet been able to arrive at these. It is because of this failure that Professor E. W. Scripture clearly points out the absolute necessity for a further work in *vowel analysis*; also, it is on account of this failure alone that certain phonemes are taken to be diphthongs which in reality turn out to be only long vowels in which the change is considerable.[2] Our normal expectation is that V and C must be mutually exclusive, *i.e.*, no sound-profiles can have both V and C. It is clearly seen from the phenomenon of the occurrence of the phoneme traditionally known as the *Aytam* in certain speech-forms in Tamil,[3] that V and C are not *exhaustive* characters of speech-sound-profiles. It is not true that a sound-profile must be either V or C. The *Aytam* is both not—V and not—C.

In the speech-forms where the *Aytam* occurs, it is preceded by a vowel (*the necessary condition*) and followed by a consonant (*the sufficient condition*). That is, in the actual articulation of certain Tamil words in the stream of speech, consisting of an integral part with a vowel, followed by a consonant, the *Aytam* occurs. In each such word, the sound-profiles which occur successively may be taken as an infinite class, densely ordered, which has initially the vowel-character, then a transitional character, and finally the consonantal character. For, a vowel is made up of a series of adjacent vibration profiles[4] the analysis of which show that all the frequencies from zero to infinity are present to a greater or less degree. 'The profile is, therefore, not a sum of a few discrete free vibrations as ordinarily supposed but an integration of an infinite number of such vibrations differing infinitely little from one another.'[5] Any spoken language consists of a succession of speech-sounds more or less overlapped[6] and these sounds can be grouped on the principle of similarity. The view that the speech is made up of a series of independent elements is erroneous.[7] 'Not only must we say that every individual sound changes from beginning to end but we must assert that each one develops out of the preceding sound and into the following one. There are no well-defined limits between neighboring sounds—not only because the limits are vague, but also because there are no independent sounds to be limited.'[8]

In this connection attention must be drawn to Professor Scripture's analysis of 'to race' (in plate VIII facing page 40). 'As the [t]-closure is opened, the vibrations appear in line 102. Are these weak vibrations to be reckoned to the vowel vibrations that occupy the rest of the line? Or are they to be treated as a "glide" from [t] to the vowel?' These are legitimate questions. But Professor Scripture does not appear to answer them. On the contrary, he appears to be merely to follow an arbitrary procedure in placing the beginning of the vowel at the point where the vibrations reach half a millimeter amplitude. Similar is his treatment with regard to other sounds as well (page 44). He himself admits that nowhere in the whole sequence is there any sudden change, nowhere any possibility of assigning limits. 'We must conclude that there are no such limits and that the sound changes gradually throughout.'

In this connection, it is interesting also to note that according to A. Tanakadate[9] the record of any single syllable of the Japanese sound elements, *e.g.*, that represented by the