LETTERS TO THE EDITOR

DISTRIBUTION OF FISHER'S g_1 FOR SAMPLES OF THREE FROM A CONTINUOUS RECTANGULAR DISTRIBUTION

AN easy method to find out the distribution of Fisher's g_1 for samples of three from continuous populations appears possible from the following considerations.

Let x_1, x_2, x_3 be the sequence in which the three individuals are observed and let

$$\tau_i=\frac{x_i-\bar{x}}{s} \text{ where } \bar{x}\ \ \tfrac{1}{3}\sum_{i=1}^{3} x_i$$

$$\text{and } s^2=\tfrac{1}{3}\sum_{i=1}^{3}(x_i-\bar{x})^2. \qquad (1)$$

It is easy to see that if the τ_i's are arranged in ascending order of absolute magnitude forming the sequence $\tau_1', \tau_2', \tau_3'$

$$\text{then } 0\leqslant|\tau_1'|\leqslant\frac{1}{\sqrt{2}};\ \frac{1}{\sqrt{2}}\leqslant|\tau_2'|\leqslant\sqrt{\frac{3}{2}};$$

$$\sqrt{\frac{3}{2}}\leqslant|\tau_3'|\leqslant\sqrt{\ }. \qquad (2)$$

Also since

$$\sum_{i=1}^{3}\tau_i'=0;\ \sum_{i=1}^{3}(\tau_i')^2=3$$

$$\text{and } \sum_{i=1}^{3}(\tau_i')^3=\sqrt{\frac{3}{2}}\,g_1 \qquad (3)$$

we find that

$$g_1=\sqrt{6}\,\tau_i'\,(\tau_i'^2-\tfrac{3}{2}) \qquad (4)$$

and the distribution of g_1 can be determined once the distribution of τ_i' is known.

From the principles outlined elsewhere,[1,3] the distribution of τ_i' follows from the distribution of τ_i.

Also

$$\tau_i=\frac{\sqrt{2}\,\theta}{\sqrt{3+\theta^2}} \qquad (5)$$

where

$$\theta=\frac{x_i-\bar{x}'}{s'} \qquad (6)$$

$\bar{x}'$ and s' are respectively the mean and the standard deviation of the two observations left after excluding x_i. Hence if the probability distribution of θ is determined that of g_1 will follow immediately. This method is applied below to get the distribution of g_1 for samples of three from a continuous rectangular distribution. The notation used above has been kept up all through this note.

RECTANGULAR DISTRIBUTION

Let

$$\left.\begin{aligned} p(x_i)&=1 \text{ for } 0\leqslant x_i\leqslant 1\\ &=0 \text{ elsewhere}\end{aligned}\right\} \qquad (7)$$

So

$$p(x_i,\bar{x}',s')\ \ 4$$

$$\left.\begin{aligned}&\text{for } 0\leqslant x_i\leqslant 1\\ &0\leqslant s'\leqslant\tfrac{1}{2};\ s'\leqslant x'\leqslant 1-s'\end{aligned}\right\} \qquad (8)$$

Also

$$p(\theta,\bar{x}',s')\ \ 4s'$$

$$\left.\begin{aligned}&\text{for } -1\leqslant\theta\leqslant 1;\ 0\leqslant s'\leqslant\tfrac{1}{2};\ s'\leqslant x'\leqslant 1-s'\\ &\text{for } 1\leqslant\theta\leqslant\infty;\ 0\leqslant s'\leqslant\frac{1}{1+\theta};\\ &\qquad s'\leqslant\bar{x}'\leqslant 1-\theta s'\\ &\text{for } -\infty\leqslant\theta\leqslant -1;\ 0\leqslant s'\leqslant\frac{1}{1-\theta};\\ &\qquad -\theta s'\leqslant x'\leqslant 1-s'\end{aligned}\right\} \qquad (9)$$

$$\left.\begin{aligned}\therefore\ p(\theta)&=\tfrac{1}{6} \text{ for } -1\leqslant\theta\leqslant 1\\ &=\tfrac{2}{3}\cdot\frac{1}{(1+\theta)^2} \text{ for } 1\leqslant\theta\leqslant\infty\\ &=\tfrac{2}{3}\cdot\frac{1}{(1-\theta)^2} \text{ for } -\infty\leqslant\theta\leqslant -1\end{aligned}\right\} \qquad (10)$$

Since

$$g_1=\frac{\sqrt{3}\,\theta\,(\theta^2-9)}{(3+\theta^2)^{3/2}} \qquad (11)$$

it follows that

$$p(g_1)=\frac{(3+\theta^2)^{5/2}}{27\sqrt{12}\,(1-\theta^2)} \text{ for } -1\leqslant\theta\leqslant 1 \qquad (12)$$