LETTERS TO THE EDITOR

TACTICAL CONFIGURATIONS AND THE METHOD OF SYMMETRIC DIFFERENCES

A tactical configuration may be defined as a finite aggregate of sets of elements combined according to given symmetrical principles. It is no wonder that such configurations occur almost everywhere in mathematics. Fisher and Yates[1] have now discovered a new use for them in the design of agricultural experiments. Various methods have been devised for the construction of particular types of configurations, centering round finite geometries, marks in the Galois field, multiply-transitive groups, and more recently the so-called symmetric differences. A little history of this last method is worth giving in this connection.

About 1931, when I struck upon a method of differences to set up finite geometries and discovered a few related theorems, I wrote to Professor Veblen, then of Princeton University, to give me some references. I quote below a few lines from his letter, dated October 7, 1931:

> "With regard to the questions raised in your letter, I would say that I think you will find the theorem mentioned in your question number 2, in some of the books on Combinatorial Mathematics, for example, in Netto's book. At any rate I remember that I used the method of differences to set up finite spaces before I knew about the Galois field method.
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> "In general, I have no doubt that there are a great many finite spaces which are not obtainable by the use of Galois field method. One of my colleagues constructed examples of such spaces by laborious trial and error methods long ago. It would be interesting to get something systematic on this subject."

Not satisfied with the above vague reply, I announced my method of differences in a paper on "Finite Geometries" read before the Nineteenth Session of the Indian Science Congress, Bangalore, 1932. On page 105 of the Abstracts,[2] mention is made of the existence of modular sets of the type $(X_1, X_2, \ldots X_n)$: mod. (n^2-n+1), where $X_r < X_{r+1}$ and the differences $X_r - X_s$, mod. (n^2-n+1), are all distinct. Such sets may be briefly referred to as modular difference sets. For example, the set (1, 2, 4, 9, 13, 19); mod. 31 gives rise to a finite projective geometry of 31 points and 31 lines with six points in each line, and six lines through each point. Statistically, this represents a balanced incomplete block design of six experimental units, with thirty-one blocks, and thirty-one varieties.

In 1938, James Singer announced a method of obtaining modular difference sets from the

Galois field theory and utilised it to give a compact combinatorial representation of a plane finite projective geometry—just the same representation that I had given seven years previously and about which Veblen had remarked dubiously that it might be found in books on Combinatorial Analysis, like Netto's. Netto's book is unfortunately not accessible to me and in the recent articles of R. C. Bose, which refer to Netto's Combinatory Analysis,[4] Veblen's Finite Geometries and Singer's difference properties,[3] there is no indication of the method of differences being specifically employed in Netto's work.

A general method of discovering all possible difference sets is yet to be devised, while the problem whether every finite geometry or randomised block arrangement implies a modular difference set, is unsolved, though the converse is now known to be true. All the methods so far known about the construction of tactical configurations involve somewhere an element of guess work which seems to be unavoidable. However, the recent collections of examples of configurations due to Archibald, Fisher, Yates, Stevens, Bose and others provide some basis for the outlines of a general theory.

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1. Fisher and Yates, "Statistical Tables for Biological, Agricultural and Medical Research," 1938.
2. "Proceedings of the Nineteenth Indian Science Congress," Bangalore, 1932.
3. Bose, R. C., *The Journal of the Indian Mathematical Society*, 1942, **6**, 1.
4. —, *Annals of Eugenics*, 1939, **9**, 4.

PHOTOELECTRIC EFFICIENCIES OF SOME METALS IN THE SOFT X-RAY REGION

The photoelectric efficiencies of some metals in the soft X-ray region were studied by Davies[1] and Bandopadhyaya[2]. The latter author investigated the efficiencies of 11 elements for soft X-rays excited at 400 volts from a copper target. The order of efficiency was found to be very similar to that under ultra-violet light.

This resemblance between the photoelectric properties due to soft X-rays and ultra-violet light has also been suggested from the similarity in the absorption coefficients in the 500-volt soft X-ray region and in the ultra-violet region, found by Rudberg[3] and Partzsch and Hallwachs[4] respectively.

In the present investigation, this similarity has been extended by a study of the photoelectric efficiencies of Zn, Cd, Sn, Pb, Al, Mg, Ag, Ni, Co and Fe. The last few metals were studied so that the results may be compared with those of previous workers.

Millikan and Winchester[5] have studied the photoelectric efficiencies of some of these metals for ultra-violet light and gave the order Pb, Zn, Mg, Al, Fe, Ag and Ni.

Soft X-rays produced from a thoroughly outgassed nickel plate and freed from ions by a condenser arrangement, were allowed to fall on freshly degassed metal surfaces for the study of their photoelectric efficiencies. Four metals were used in one setting, these being arranged in the form of a lantern, with the degassing filament enclosed within the structure. The photoelectrons liberated were attracted to an external shield raised to a positive potential and the photoelectric current was measured by allowing it to leak through a high resistance, one end of the resistance being earthed and the other end being connected to a quadrant electrometer. The high resistance was made of a smoked quartz rod and had a resistance of about 10^{11} ohms. The experiments were carried out in high vacuum, the pressure inside the pyrex working tube being much lower than 10^{-6} mm. of mercury.

The metals could be arranged in the following order of decreasing photoelectric sensitivity:—Zn, Cd, Sn, Pb, Al, Mg, Ag, Ni, Co and Fe. This agrees well with the order given above by Millikan and Winchester[5] for some of these metals.

The following figures give the values of the work function of the metals, wherever these