

dilute solutions, as described in the previous papers published by the author.<sup>1,2,3</sup>

More work is in progress to arrive at an explanation of the above results.

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<sup>1</sup> N. R. Rao, *Ind. Journ. Phys.*, 1940, **14**, 143.

<sup>2</sup> —, *Ibid.*, 1941, **15**, 185.

<sup>3</sup> —, *Ibid.*, 1942, **16**, 71.

### DARCY'S LAW AND UPWARD MOVEMENT OF WATER IN SOIL

IN an interesting paper Ramdas and Malik<sup>1</sup> give an account of their recent studies on the upward movement of water and salt solutions in the black cotton soil. The rate of ascent of water in vertical glass tubes packed with soil was determined. The rate of ascent is at first rapid and then gradually falls off with time.

It seems worthwhile to point out in this connection that the relation between  $h$  and  $t$ , where  $h$  denotes the height upto which water rises in the tube in time  $t$ , can be immediately derived by an obvious application of Darcy's Law governing the flow of a fluid in a porous medium and the experiments of Ramdas and Malik can be used to find the permeability coefficient of the soil. Let  $h_0$  denote the maximum height to which water rises in the tube (i.e.,  $h \rightarrow h_0$  as  $t \rightarrow \infty$ ), then the pressure forcing the water upwards at time  $t$ , when the height of the water level in the tube is  $h$ , will be  $(h_0 - h)g\rho$ ,  $\rho$  being the density of water. Hence according to Darcy's Law the velocity  $v$  of flow will be given by

$$v = \frac{dh}{dt} = \frac{K}{\mu} \frac{(h_0 - h) g \rho}{h}, \quad (1)$$

$$\text{or } t = \frac{\mu h_0}{K g \rho} \left\{ \log \frac{h_0}{h_0 - h} - \frac{h}{h_0} \right\}, \quad (2)$$

where  $K$  is called the permeability coefficient, and  $\mu$  is the viscosity of water. (It may be noted that for the flow of water in a capillary tube of radius  $a$ ,  $K$  will be  $\frac{a^2}{8}$ ). The experimental curve of Ramdas and Malik agrees with

the relation (2) for  $K = (4.68 \pm 5)$  milli-Darcys. These experiments, therefore, provide a method for determining the permeability coefficient of soils.

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<sup>1</sup> L. A. Ramdas and A. K. Malik, *Proc. Ind. Acad. of Sci.*, 1942, **16**, 1.

<sup>2</sup> Musket, *The Flow of Homogeneous Fluids through Porous Media*, Chapter II (McGraw Hill, 1937).

### COEFFICIENT OF EXPANSION OF SOLIDS

THE model of the solid body which is the basis of the atomic heat theories of Einstein, Debye and Born-Karman is highly idealised. This idealised solid body has, as is easily seen, a zero coefficient of expansion. In order to explain the expansion of a solid, Debye assumed a law of force involving higher powers of the variation of the atomic distance. This extension gives the law deduced by Gruneisen<sup>1</sup> that at sufficiently low temperatures the coefficient of thermal expansion  $\alpha$  is proportional to the specific heat  $C_v$ .

The coefficient of volume expansion  $\alpha$  is given by the relation

$$\alpha = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_P = \beta_\theta \left( \frac{\partial}{\partial V} \right)_T \int_0^T C_v \frac{dT}{T}.$$

If we assume that  $C_v$  is a function of  $\frac{\Theta}{T}$ , where  $\Theta = \frac{h\nu}{k}$  after Debye or Einstein, then

$$\begin{aligned} \alpha &= - \frac{\beta_\theta}{V} \frac{\partial \log \Theta}{\partial \log V} C_v \\ &= \frac{\beta_\theta}{V} C_v \gamma, \end{aligned}$$

where  $\beta_\theta =$  isothermal compressibility

$$\gamma = - \frac{\partial \log \Theta}{\partial \log V}.$$

It is well known that the Debye formula fails to represent the specific heat curves correctly in several cases. Elaborate hypothesis have been put forward to explain away these failures. According to Raman<sup>2</sup> the vibration

spectrum of every crystal, however simple in its chemical composition, includes several monochromatic frequencies lying in the infra-red or high frequency region. This is accompanied in the region of low frequencies by a continuous spectrum of elastic vibrations. The contribution of the latter to the thermal energy would be relatively of minor importance for the vast majority of crystals except at the lowest temperatures. When these monochromatic frequencies are known, the specific heat formula necessarily contains corresponding Einstein terms. Thus in the specific heat theory developed by Raman

$$C_{\nu} = w_1 C_{\nu 1} + w_2 C_{\nu 2} + w_3 C_{\nu 3},$$

where  $w_1, w_2, w_3$  are the weight factors depending on the nature of the crystal and  $C_{\nu 1}, C_{\nu 2}$  and  $C_{\nu 3}$  are Einstein functions corresponding to the monochromatic frequencies  $\nu_1, \nu_2, \nu_3$ , etc. On the basis of this theory the coefficient of volume expansion of a crystal is given by

$$\alpha = - \frac{\beta \theta}{V} \left\{ w_1 C_{\nu 1} \frac{\partial \log \Theta_1}{\partial \log V} + w_2 C_{\nu 2} \frac{\partial \log \Theta_2}{\partial \log V} + w_3 C_{\nu 3} \frac{\partial \log \Theta_3}{\partial \log V} \right\}$$

The factors  $\frac{\partial \log \Theta}{\partial \log V}$  give the dependence of the vibrational frequencies on the volume of the crystal. For quasi-harmonic vibrations it is an index of the relationship between the amplitude of vibrations and the volume of the crystal. The factors  $C_{\nu i}$ , etc., give the temperature-rate at which the crystal takes up energy, i.e., the amplitude of vibration increases. The product of these two factors is then an index of the temperature-rate of volume change. We will assume that each  $\frac{\partial \log \Theta}{\partial \log V}$  is a temperature independent quantity. This assumption is reasonable as long as the anharmonicity is small. If all the frequencies show the same volume dependence then  $\frac{\partial \log \Theta}{\partial \log V}$  is the same for all of the frequencies. In such a case it reduces to the expression deduced on the basis of Debye's theory. But actually  $\nu_1, \nu_2$  and  $\nu_3$  differ widely in character and their anharmonicities may be supposed to differ just as widely. In such a case the

factors  $\frac{\partial \log \Theta}{\partial \log V}$  will not be the same and we can write

$$a = a C_{\nu 1} + b C_{\nu 2} + c C_{\nu 3}, \text{ where}$$

$$a = - w_1 \frac{\beta \theta}{V} \cdot \frac{\partial \log \Theta_1}{\partial \log V}, \quad b = - w_2 \frac{\beta \theta}{V} \frac{\partial \log \Theta_2}{\partial \log V},$$

$$c = - w_3 \frac{\beta \theta}{V} \frac{\partial \log \Theta_3}{\partial \log V}.$$

Since  $\frac{\beta \theta}{V}$  varies very little with temperature, we can regard  $a, b, c$  as temperature independent quantities. From a knowledge of  $a, b$  and  $c, \alpha$  could be calculated. Unfortunately there appears at present to be no way of obtaining them. Hence the above equation can be used in a purely empirical fashion by evaluating  $a, b$  and  $c$  from the observed  $\alpha$  values over which the observations of  $\alpha$  is free from doubt. If we assume the thermal expansion  $\alpha$  to be resolvable into three components, the three constants  $a, b$  and  $c$  can be evaluated from  $C_{\nu 1}, C_{\nu 2}$  and  $C_{\nu 3}$  and  $\alpha$  at any three temperatures. It should then be possible to represent  $\alpha$  as a function of temperature over the whole range of the crystal. The values calculated in this way for lithium are included in the following table. Similar results hold in the case of aluminium, lead and silver. Other details will be published elsewhere.

Lithium,  $a = 19.69, b = -5.71, c = 29.7$ .

Temp. °A	Calculated Value of $\alpha \times 10^6$	Observed Value of $\alpha \times 10^6$
15°	0.062	no obsn.
60	4.90	no obsn.
90	15.01	no obsn.
100	18.49	18.50
120	24.85	24.45
140	29.32	29.60
200	39.58	39.70
260	45.38	45.00
280	46.19	46.20

In conclusion it gives me great pleasure to record my thanks to Prof. Sir C. V. Raman, kt., F.R.S., N.L., for his valuable help.

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<sup>1</sup> *Statistical Thermodynamics*, by Fowler and Guggenheim.

<sup>2</sup> Raman, *Proc. Ind. Acad. Sci., A*, Sep. 1941,