

postulate; the geodesics of the field of m_1 and m_2 can also be obtained in the limiting case $m_1 = 0$. The question in which one is interested is this. Will the equations of motion for the case $m_1 = 0$ as derived from (2) be identical with the corresponding equations derived from the geodesic postulate applied to the field satisfying (1)? If one studies the procedure of Einstein and his collaborators there is nothing to indicate that the two should be identical; and in fact their work is guided by the supposition that the two results need not be identical. On carrying out the necessary calculations we obtain the surprising result that the equations of motion of Einstein's new relativity such as (2) are fully in accord with the geodesic postulate at least up to the second order of the masses. The calculations in question are lengthy and they will be published elsewhere. It looks as if the result is not accidental for the number of terms involved in the equations is large. The two methods of deriving the equations, although so different apparently, might be logically interconnected.³

V. V. NARLIKAR.

Department of Mathematics,
Benares Hindu University,
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¹ Einstein, Infeld and Hoffmann, *Ann. Math.*, 1938, 65, 5, 39.

² Narlikar, V. V., *J. Bombay Univ.*, 1939, 51, 8.

³ Narlikar, V. V., and Singh, J., *Phil. Mag.*, 1937, 628, 23.

STANDARD ERROR OF THE DIFFERENCE BETWEEN TWO ESTIMATES FOR INCOMPLETE BLOCK EXPERIMENTS

THE calculation of the standard error for comparing two treatment estimates in the case of simple experiments, like randomized blocks or Latin squares, is easy and is equal to $\sqrt{2s^2/n}$, where s^2 and n are the residual variance and the number of times each treatment is repeated in the experiment. But in designs involving incomplete blocks, the algebraic expression giving the treatment differences will have to be written

down for calculating their standard error. This is a very laborious and cumbersome procedure. A simple method for calculating the standard error of the difference between two treatment estimates for any experiment is given below:

First we determine the residual error of the whole experiment by subtracting the reduction in the sum of squares for blocks and treatments from the total sum of squares. To obtain now the standard error for the difference between any two treatments, calculate the sum of squares for the difference between the two treatments, as explained in a previous paper,¹ by subtracting the reduction in the sum of squares for blocks and treatments, assuming that there is no difference between the two treatments in question, from the sum of squares for blocks and treatments which has been determined before. Let this difference be A and the residual variance be s^2 . It can be now shown that the standard error for the difference between the two treatments is equal to

$$\frac{s(t_1 - t_2)}{\sqrt{A}},$$

where t_1 and t_2 are the least square estimates of the treatments.

In the case of balanced incomplete blocks experiments, it is easy to see that the standard error for the difference between any two treatments is the same. But for asymmetrical experiments, this will be different for different differences.

P. V. KRISHNA IYER.

Imperial Agricultural Research Institute,
New Delhi,
January 14, 1941.

¹ *Proc., Ind. Acad. Sci.*, 11, 369.

"EXPECTATION" OF GROWTH OF POPULATION

IN the *Indian Journal of Economics* of June 1940, Mr. D. Sen Gupta obtains the formula

$$y - d = \frac{k}{1 + ce^{rt}} \quad \dots \quad (A)$$

where y is the population, t is the time measured from a base year and c , d , k and r are

constants for extrapolating for population figures where birth, death and migration statistics are not sufficiently reliable.

2. It will simplify the notation if instead of t we write $10t$ as the time measured in years from the last census, and the figures for the decennial censuses in reverse order as a_0, a_1, a_2, \dots etc., but the extrapolation formula is only required for intercensal use and we may, therefore, suppose $0 < t < 1$.

It is easy to see that a differential equation

$$\frac{1}{y} \frac{dy}{dt} = k(A - y) \quad \dots \quad (1)$$

where k and A are constants, connecting the proportional rate of growth of population with the amount by which the population at the time falls short of a constant number leads to the solution

$$y = \frac{A}{1 + e^{-Akt - c}} \quad \dots \quad (2)$$

where c is a fresh constant, which differs from (A) only in that it involves three and not four constants. It is easy to see that if the formula (1) holds the population is always above or always below A .

The constants involved can be evaluated in terms of a_0, a_1 and a_2 and we obtain a formula very similar to that of Mr. Griffiths referred to in the same paper.

3. To obtain an estimate of the effects of faulty computations of the constants A and k we differentiate logarithmically the formula

$$\log \left(\frac{A - y}{A - a_0} \cdot \frac{a_0}{y} \right) = -Akt \quad \dots \quad (3)$$

obtained by using the fact that when $t = 0$ $y = a_0$.

4. Even if the A and k of the equation (1) change with time, but their changes in a period of thirty years are comparatively small, the population obtained from (3) would be fairly reliable. A and k will be characteristic of the population studied and if the formula is found to hold, the slow secular changes in these quantities should be of great interest.

5. A formula of this nature assumed to hold for comparatively short periods avoids the recent criticism of such sociological laws as Pareto's of claiming an improbable degree of

universality. In passing, we observe that Pareto's Law is given an appearance of greater plausibility if we consider it as giving the "expectation" of receiving an income large in comparison with the average income of the group.

H. E. PERIES.

The Chief Secretary's Office,
Colombo, Ceylon,
February 6, 1940.

ARC DISCHARGE IN MERCURY

THE difference of potential across the electrodes of a mercury arc lamp in vacuum varies almost linearly with current, showing that here Ohm's Law holds good. It has been shown by Henri¹ that the voltage-current characteristic curve, however, showed a slight concavity towards the current axis for large values of current, when the arc was cooled by immersing the arc lamp under water.

We have been able to get a large number of characteristics for a sealed arc, at a pressure of about 0.5 mm., some of which are reproduced below (Fig. 1). The arc is cooled by blowing a current of air with an electric fan. By varying the distance of the fan, the lamp can be cooled to different degrees.

Curve I is a straight line characteristic. Curve II shows a negative or falling part in the characteristic as obtained when the lamp is cooled with the fan running at a distance of 4 metres. The succeeding curves show the falling parts more and more prominently as the cooling is increased. The falling part of the characteristic does not seem to have been observed by previous workers for mercury arcs.

The above curves show a minima of about 23.5 volts which appears to be almost independent of the degree of cooling. For this voltage the arc fills the whole of the cathode surface, while for voltages on the falling part of the characteristic the surface is only partially filled.