

Einstein's universe: The challenge of dark energy

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From an observational perspective, cosmology is today in excellent shape – we can now look back to when the first galaxies formed ~ 1 Gyr after the Big Bang, and reconstruct the thermal history back to the primordial nucleosynthesis era when the universe was ~ 1 s old. However recent deep studies of the Hubble diagram of Type Ia supernovae indicate that the expansion rate is accelerating, requiring the dominant component of the universe to have *negative* pressure like vacuum energy. This has been indirectly substantiated through detailed studies of angular anisotropies in the cosmic microwave background and of spatial correlations of the large-scale structure of galaxies, which also require most of the matter component to be non-baryonic. Although there are plausible candidates for the constituent of the dark matter in physics beyond the Standard Model (e.g. supersymmetry), the energy scale of the required ‘dark energy’ is $\sim 10^{-12}$ GeV, well below any known scale of fundamental physics. This has refocused attention on the notorious cosmological constant problem at the interface of general relativity and quantum field theory. It is likely that the resolution of this issue will require fundamental modifications to Einstein's ideas about gravity.

Keywords: Cosmic microwave background, dark energy, dark matter, Hubble expansion, relativistic cosmology.

1. Introduction

In the accompanying article, Jayant Narlikar recounts how in 1917 Einstein boldly applied his newly developed theory of general relativity to the universe as a whole¹. The first cosmological model was *static* to match prevalent ideas about the universe (which, at that time, was confined to the Milky Way!) and to achieve this Einstein introduced the ‘cosmological constant’ term in his equation². Within a decade it had become clear from the work of Slipher and Hubble that the nebulae on the sky are in fact other ‘island universes’ like the Milky Way and that they are receding from us – the universe is expanding. Einstein wrote to Weyl in 1933: ‘If there is no quasi-static world, then away with the cosmological term’.

This however was not easy – after all the symmetry properties of Einstein's equation do allow any constant proportional to the metric to be added to the left hand side:

$$R_{mm} - \frac{1}{2} g_{mm} R + \Lambda g_{mm} = \frac{-8pT_{mm}}{M_p^2}, \quad (1)$$

where we have written Newton's constant, $G_N \equiv 1/M_p^2$ (where $M_p \simeq 1.2 \times 10^{19}$ GeV), in natural units ($\hbar = k_B = c = 1$). Moreover with the subsequent development of quantum field theory it became clear that the energy-momentum tensor on the right hand side can also be freely scaled by another additive constant proportional to the metric which reflects the (Lorentz invariant) energy density of the vacuum:

$$\langle T_{mm} \rangle_{\text{fields}} = -\langle \mathbf{r} \rangle_{\text{fields}} g_{mm}. \quad (2)$$

This contribution from the matter sector adds to the ‘bare’ term from the geometry, yielding an effective cosmological constant:

$$\Lambda = \Lambda + \frac{8p\langle \mathbf{r} \rangle_{\text{fields}}}{M_p^2}, \quad (3)$$

or, correspondingly, an effective vacuum energy: $\mathbf{r}_v \equiv \Lambda M_p^2/8p$.

For an (assumed) homogeneous and isotropic universe with the Robertson–Walker metric $ds^2 \equiv g_{mn} dx^m dx^n = dt^2 - R^2(t)[dr^2/(1 - kr^2) + r^2 d\Omega^2]$, we obtain the Friedmann equations describing the evolution of the cosmological scale-factor $R(t)$:

$$H^2 \equiv \left(\frac{\dot{R}}{R} \right)^2 = \frac{8p}{3M_p^2} \mathbf{r} - \frac{k}{R^2} + \frac{\Lambda}{3},$$

$$\frac{\ddot{R}}{R} = \frac{4p}{3M_p^2} (\mathbf{r} + 3\mathbf{r}) + \frac{\Lambda}{3}, \quad (4)$$

where $k = 0, \pm 1$ is the 3-space curvature signature and we have used for ‘ordinary’ matter (and radiation): $T_{mm} = pg_{mm} + (p + \mathbf{r})u_m u_n$, with $u_m \equiv dx_m/ds$. The conservation equation $T_n^m = 0$ implies that $d(\mathbf{r}R^3)/dR = -3pR^2$, so that given the ‘equation of state parameter’ $w \equiv p/\mathbf{r}$, the evolution history can be constructed. Since the redshift is $z \equiv R/R_0 - 1$, for non-relativistic particles with $w \simeq 0$,

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$\mathbf{r}_{\text{NR}} \propto (1+z)^{-3}$, while for relativistic particles with $w = 1/3$, $\mathbf{r}_{\text{R}} \propto (1+z)^{-4}$, but for the cosmological constant, $w = -1$ and $\mathbf{r}_{\text{v}} = \text{constant}$! Thus radiation was dynamically important only in the early universe (in fact for $z \gtrsim 10^4$) and for most of the expansion history only non-relativistic matter is relevant. The Hubble equation can then be rewritten with reference to the present epoch (subscript 0) as

$$H^2 = H_0^2[\Omega_{\text{m}}(1+z)^3 + \Omega_{\text{k}}(1+z)^2 + \Omega_{\Lambda}],$$

$$\Omega_{\text{m}} \equiv \frac{\mathbf{r}_{\text{m}0}}{\mathbf{r}_{\text{c}}}, \quad \Omega_{\text{k}} \equiv -\frac{k}{R_0^2 H_0^2}, \quad \Omega_{\Lambda} \equiv \frac{\Lambda}{3H_0^2}, \quad (5)$$

yielding the sum rule $\Omega_{\text{m}} + \Omega_{\text{k}} + \Omega_{\Lambda} = 1$. Here $\mathbf{r}_{\text{c}} \equiv 3H_0^2 M_{\text{P}}^2 / 8\pi \simeq (3 \times 10^{-12} \text{ GeV})^4 h^2$ is the ‘critical density’ for a $k = 0$ universe (in the absence of Λ) and the present Hubble parameter is $H_0 \equiv 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$ with $h \simeq 0.7$, i.e. about 10^{-42} GeV .

As Weinberg discussed in his influential review in 1989 (ref. 3), given that the density parameters Ω_{m} and Ω_{k} were observationally known to be not much larger than unity, the two terms in eq. (3) are required to somehow conspire to cancel each other in order to satisfy the approximate constraint

$$|\Lambda| \lesssim H_0^2, \quad (6)$$

thus bounding the present vacuum energy density by $\mathbf{r}_{\text{v}} \lesssim 10^{-47} \text{ GeV}^4$ which is a factor of 10^{123} below its ‘natural’ value of $\sim M_{\text{P}}^4$ – the cosmological constant problem! The major development in recent years has been the recognition that this inequality is in fact saturated with $\Omega_{\Lambda} \simeq 0.7$, $\Omega_{\text{m}} \simeq 0.3$ ($\Omega_{\text{k}} \simeq 0$) (ref. 4), i.e. there *is* non-zero vacuum energy of just the right order of magnitude so as to be detectable today.

In the Lagrangian of the Standard Model of electro-weak and strong interactions, the term corresponding to the cosmological constant is one of the two ‘super-renormalizable’ terms allowed by the gauge symmetries, the second one being the quadratic divergence in the mass of fundamental scalar fields due to radiative corrections⁵. To tame the latter sufficiently in order to explain the experimental success of the Standard Model has required the introduction of a supersymmetry between bosonic and fermionic fields which is (softly) broken at about the Fermi scale. Thus the cutoff scale of the Standard Model, viewed as an effective field theory, can be lowered from the Planck scale M_{P} down to the Fermi scale, $M_{\text{EW}} \sim G_{\text{F}}^{-1/2} \sim 300 \text{ GeV}$, albeit at the expense of introducing about 150 new parameters in the Lagrangian, as well as requiring delicate control of the many non-renormalizable operators which can generate flavour-changing neutral currents, nucleon decay, etc. so as not to violate experimental bounds. This implies a *minimum* contribution to the vacuum energy density from quantum fluctuations of $\mathbf{O}(M_{\text{EW}}^4)$, i.e. ‘halfway’ on a logarithmic scale down from the Planck

scale to the energy scale of $\mathbf{O}(M_{\text{EW}}^2/M_{\text{P}})$ corresponding to the observationally indicated vacuum energy. Thus even the introduction of supersymmetry cannot eradicate a discrepancy by a factor of at least 10^{60} between ‘theory’ and observation.

It is generally believed that a true resolution of the cosmological constant problem can only be achieved in a full quantum theory of gravity. Recent developments in string theory and the possibility that there exist new dimensions in Nature have generated many interesting ideas concerning possible values of the cosmological constant^{2,6}. Nevertheless it is still the case that there is *no* generally accepted solution to the enormous discrepancy discussed above. Of course the cosmological constant problem is not new but there has always been the expectation that somehow we would understand one day why it is exactly zero. However if it is in fact non-zero and dynamically important *today*, the crisis is much more severe since it also raises a cosmic ‘coincidence’ problem, viz. why is the present epoch special? It has been suggested that the ‘dark energy’ may not be a cosmological *constant* but rather the slowly evolving potential energy $V(\mathbf{f})$ of a hypothetical scalar field \mathbf{f} named ‘quintessence’ which can *track* the matter energy density. This however is also fine-tuned since one needs $V^{1/4} \sim 10^{-12} \text{ GeV}$ but $\sqrt{d^2V/d\mathbf{f}^2} \sim H_0 \sim 10^{-42} \text{ GeV}$ (in order that the evolution of \mathbf{f} be sufficiently slowed by the Hubble expansion), and moreover does not address the fundamental issue, viz. why are all the other possible contributions to the vacuum energy absent? Admittedly the latter criticism also applies to attempts to do away with dark energy by interpreting the data in terms of modified cosmological models. Given the ‘no-go’ theorem against dynamical cancellation mechanisms in eq. (3) in the framework of general relativity³, it might appear that solving the problem will necessarily require our understanding of gravity to be modified. However to date no such alternative which is phenomenologically satisfactory has been presented. The situation is so desperate that ‘anthropic’ arguments have been advanced to explain why the cosmological constant is just of the right order of magnitude to allow our existence today, notwithstanding the fact that we have little or no understanding of its prior probability distribution!

Given this sorry situation on the theoretical front, this article will focus solely on the new observational developments and present a critical assessment of the evidence for dark energy. It is no exaggeration to say that this tiny energy density of the present vacuum poses the biggest challenge that fundamental theory and cosmology have ever faced.

2. The observational situation

That we live in a universe which has evolved from a hot dense past is now well established, primarily on the basis

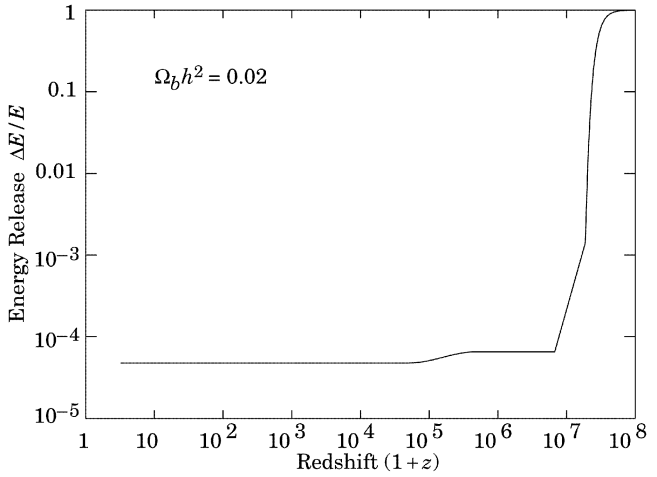
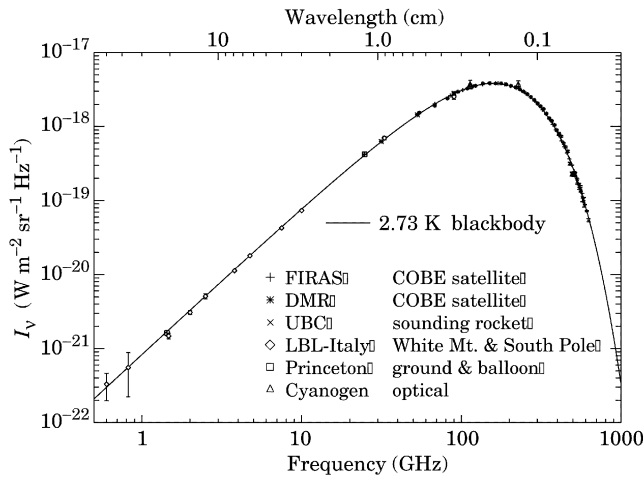


Figure 1. The spectrum of the cosmic microwave background, demonstrating the excellent fit to a blackbody; as shown in the right panel this imposes severe constraints on any deposition of entropy in the universe back to the thermalization epoch¹¹.

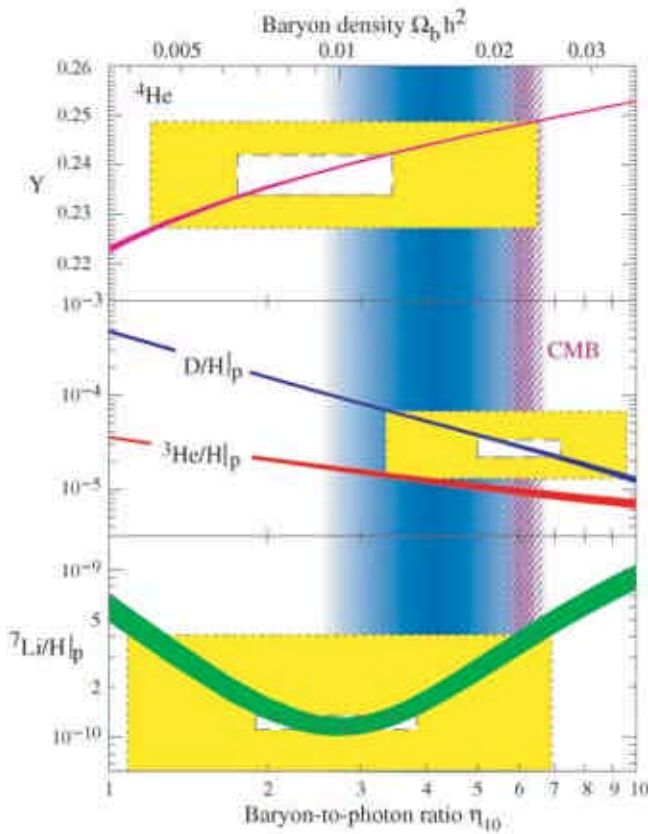


Figure 2. The abundances of ⁴He, D, ³He and ⁷Li as predicted by the standard model of big-bang nucleosynthesis as a function of $h_{10} \equiv h/10^{-10}$ (ref. 12). Boxes indicate the observed light element abundances (smaller boxes: 2σ statistical errors; larger boxes: $\pm 2\sigma$ statistical and systematic errors); the narrow vertical band indicates the CMB measure of the cosmic baryon density.

of the exquisite match (see Figure 1) of a Planck spectrum to the intensity of the cosmic microwave background (CMB), with

$$T_0 = 2.725 \pm 0.001 \text{ K}, \tag{7}$$

as determined by the COBE satellite⁷. It has also been shown that the blackbody temperature does increase with the redshift as $T = T_0 (1 + z)$, by observing fine-structure transitions between atomic levels of CI in cold gas clouds along the line of sight to distant quasars⁸. These observations are difficult to accommodate within the ‘Quasi-Steady State Cosmology’ in which the CMB arises through thermalization of starlight⁹ – a mechanism that was already severely constrained by the closeness of the observed CMB spectrum to the Planck form¹⁰. Moreover the absence of spectral distortions requires that the evolution must have been very close to adiabatic (i.e. constant entropy per baryon) back at least to the epoch when the universe was dense enough and hot enough for radiative photon creation processes to be in equilibrium (at $z \gtrsim 10^7$) (ref. 11).

A modest extrapolation in redshift back to $z \sim 10^{10}$ takes us back to the epoch of Big Bang nucleosynthesis (BBN) when the weak interactions interconverting neutrons and protons became too slow to maintain chemical equilibrium in the cooling universe, and the subsequent nuclear reactions rapidly converted about 25% of the total mass into the most stable light nucleus ⁴He. Trace amounts of D, ³He and ⁷Li were also left behind with abundances sensitive to the baryon density. As shown in Figure 2, the primordial abundances of all these elements as inferred from a range of observations are in reasonable agreement with the standard calculation¹². Moreover the implied baryon-to-photon ratio $h = n_b/n_g = 2.74 \times 10^{-8} \Omega_B h^2$ ($\Omega_B \equiv r_B/r_C$) is in good agreement with the value deduced from observations of CMB anisotropies generated at a redshift of $z \sim 10^3$ when the primordial plasma recombines and requires that baryons (more precisely, nucleons) contribute only $\Omega_B = 0.012\text{--}0.025$ h⁻², i.e. most of the matter in the universe must be non-baryonic. Moreover this concordance is an extremely powerful constraint on new physics,

e.g. it requires that, barring conspiracies, the strengths of all the fundamental interactions (which *together* determine the n/p ratio at decoupling) cannot have been significantly different (more than a few per cent) from their values today. Furthermore, the dominant energy density in the universe at that epoch must have been radiation – photons and three species of (light) neutrinos. This rules out for example the interesting possibility that there has *always* been a cosmological constant Λ of $\mathbf{O}(H^2)$, since according to the first Friedmann equation (4), this is equivalent (taking $k = 0$) to a significant renormalization of the Planck scale (i.e. Newton's constant) which would be in conflict with the observed light element abundances.

These are two of the 'pillars' that the standard Big Bang cosmology is based on and they provide a secure understanding of the thermal history back to when the universe was hot enough to melt nuclei at an age of ~ 1 s. We turn now to a detailed discussion of the third 'pillar', viz. the Hubble expansion, which has been probed back only to a redshift of $\mathbf{O}(1)$ but this of course encompasses most of the actual time elapsed since the Big Bang.

2.1 The age of the universe and the Hubble constant

Advances in astronomical techniques have enabled radioactive dating to be performed using stellar spectra. Figure 3 shows the detection of the 386 nm line of singly ionized ^{238}U in an extremely metal-poor (i.e. very old) star in the halo of our Galaxy¹³. The derived abundance, $\log(\text{U}/\text{H}) = -13.7 \pm 0.14 \pm 0.12$ corresponds to an age of 12.5 ± 3 Gyr, consistent with the age of 11.5 ± 1.3 Gyr for the (oldest stars in) globular clusters inferred, using stellar evolution models, from the observed Hertzsprung–Russell diagram¹⁴. To this must be added ~ 1 Gyr, the estimated epoch of galaxy/star formation, to obtain the age of the universe.

For the Big Bang cosmology to be valid this age must be consistent with the expansion age of the universe derived from measurement of the present Hubble expansion rate. The Hubble Space Telescope Key Project¹⁵ has made direct measurements of the distances to 18 nearby spiral galaxies (using Cepheid variables) and used these to calibrate five secondary methods which probe further; all data are consistent with $H_0 = 72 \pm 3 \pm 7 \text{ km s}^{-1} \text{ Mpc}^{-1}$, as shown in Figure 4. It has been argued, however, that the Key Project data need to be corrected for local peculiar motions using a more sophisticated flow model than was actually used, and also for metallicity effects on the Cepheid calibration – this would lower the value of H_0 to $63 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (ref. 16). Even smaller values of H_0 are also obtained by 'physical' methods such as measurements of time delays in gravitationally lensed systems, which bypasses the traditional 'distance ladder' and probes to far deeper distances than the Key Project. At present ten multiply-imaged quasars have well measured time delays; taking the lenses to be isothermal dark mat-

ter halos yields $H_0 = 48 \pm 3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (ref. 17). Measurements of the Sunyaev–Zeldovich effect in 41 X-ray emitting galaxy clusters also indicate a low value of $H_0 \sim 61 \pm 3 \pm 18 \text{ km s}^{-1} \text{ Mpc}^{-1}$ for a $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ universe, dropping further to $H_0 \sim 54 \text{ km s}^{-1} \text{ Mpc}^{-1}$ if $\Omega_m = 1$ (ref. 18). Both models imply an acceptable age for the universe, taking these uncertainties into account.

2.2 The deceleration parameter

The most exciting observational developments in recent years have undoubtedly been in measurements of the deceleration parameter $q \equiv dH^{-1}/dt - 1 = \frac{\Omega_m}{2} - \Omega_\Lambda$. This has been found to be *negative* through deep studies of the Hubble diagram of Type Ia supernovae (SNe Ia) pioneered by the *Supernova Cosmology Project*¹⁹ and the *High- z SN Search Team*²⁰. Their basic observation was that distant supernovae at $z \sim 0.5$ are $\Delta m \sim 0.25$ mag (corresponding to $10^{\Delta m/2.5} - 1 \approx 25\%$) fainter than would be expected for a decelerating universe such as the $\Omega_m = 1$ Einstein-deSitter model. This has been interpreted as implying that the expansion rate has been *speeding up* since then, thus the observed SNe Ia are actually further away than expected. Note that the measured apparent magnitude m of a source of known absolute magnitude M yields the 'luminosity distance':

$$m - M = 5 \log \left(\frac{d_L}{\text{Mpc}} \right) + 25, \quad d_L = (1+z) \int_0^z \frac{dz'}{H(z')}, \quad (8)$$

which is sensitive to the expansion history, hence the cosmological parameters. According to the second Friedmann equation (4) an accelerating expansion rate requires the dominant component of the universe to have *negative* pressure. The more mundane alternative possibility that the SNe Ia appear fainter because of absorption by intervening dust can be constrained since this would also lead to characteristic reddening, unless the dust has unusual properties²¹. It is more difficult to rule out that the dimming is due to evolution, i.e. that the distant SNe Ia (which exploded over 5 Gyr ago!) are intrinsically fainter by $\sim 25\%$ (ref. 22). Many careful analyses have been made of these possibilities and critical reviews of the data have been given²³.

Briefly, SNe Ia are observationally known to be a rather homogeneous class of objects, with intrinsic peak luminosity variations $\leq 20\%$, hence particularly well suited for cosmological tests which require a 'standard candle'²⁴. They are characterized by the absence of hydrogen in their spectra²⁵ and are believed to result from the thermonuclear explosion of a white dwarf, although there is as yet no 'standard model' for the progenitor(s)²⁶. However it is known (using nearby objects with independently known distances) that the time evolution of SNe Ia is

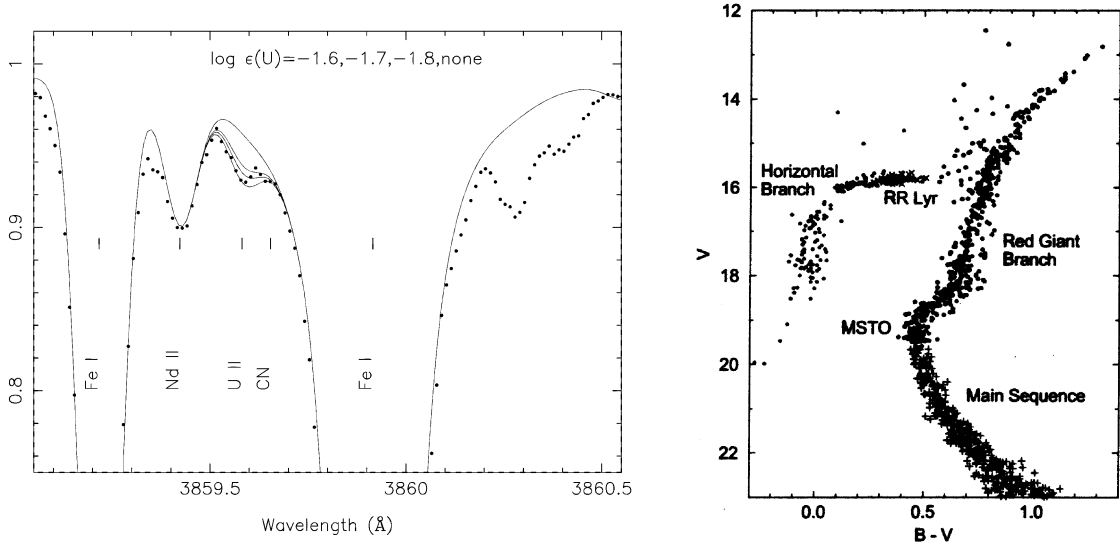


Figure 3. Detection of ^{238}U in the old halo star CS31802-001; synthetic spectra for three assumed values of the abundance are compared with the data¹³. The right panel shows the colour (B-V) versus the magnitude (V) for stars in a typical globular cluster M15; the age is deduced from the luminosity of the (marked) ‘main sequence turn-off’ point¹⁴.

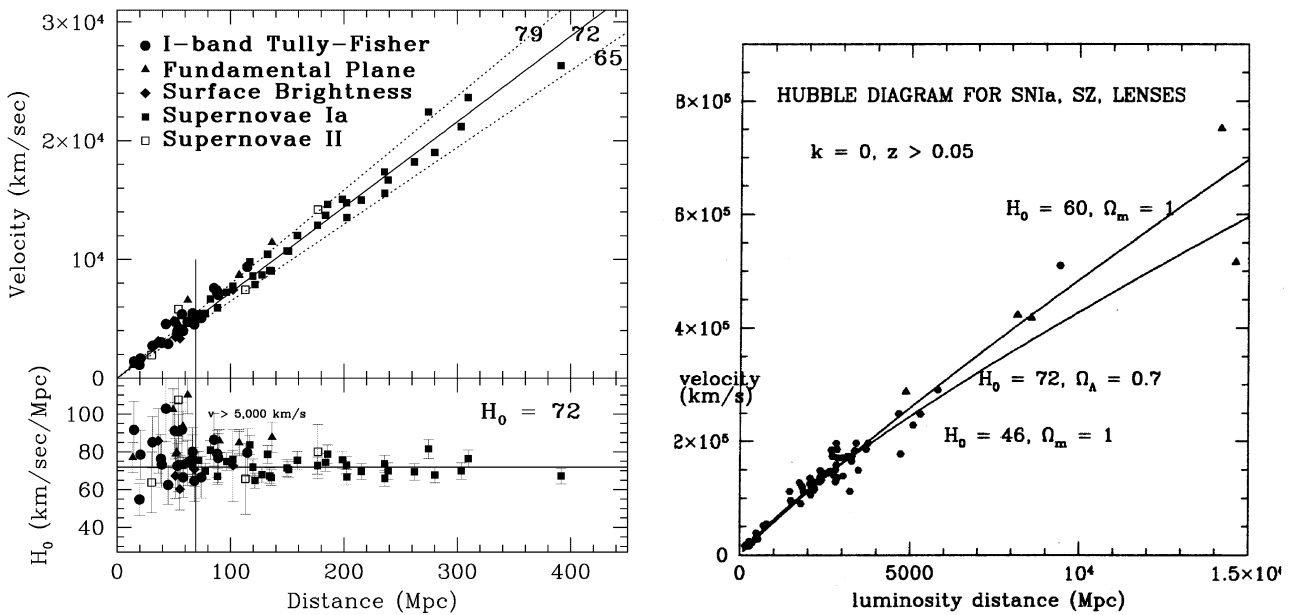


Figure 4. Hubble diagram for Cepheid-calibrated secondary distance indicators; the bottom panel shows how fluctuations due to peculiar velocities die out with increasing distance¹⁵. The right panel shows deeper measurements using SNe Ia (filled circles), gravitational lenses (triangles) and the Sunyaev-Zeldovich effect (circles), along with some model predictions⁴⁸.

tightly correlated with their peak luminosities such that the intrinsically brighter ones fade faster – this can be used to make corrections to reduce the scatter in the Hubble diagram using various empirical methods such as a ‘stretch factor’ to normalize the observed apparent peak magnitudes¹⁹ or the ‘Multi-colour Light Curve Shape’ method²⁰. Such corrections are *essential* to reduce the scatter in the data sufficiently so as to allow meaningful deductions to be made about the cosmological model.

Figure 5 shows the magnitude-redshift diagram of SNe Ia obtained recently by the *Supernova Search Team*²⁷ – this uses a carefully compiled ‘gold set’ of 142 SNe Ia from ground-based surveys, together with 14 SNe Ia in the range $z \sim 1-1.75$ discovered with the HST. The latter are *brighter* than would be expected if extinction by dust or simple luminosity evolution is responsible for the observed dimming of the SNe Ia up to $z \sim 0.5$, and thus support the earlier indication of an accelerating cosmological

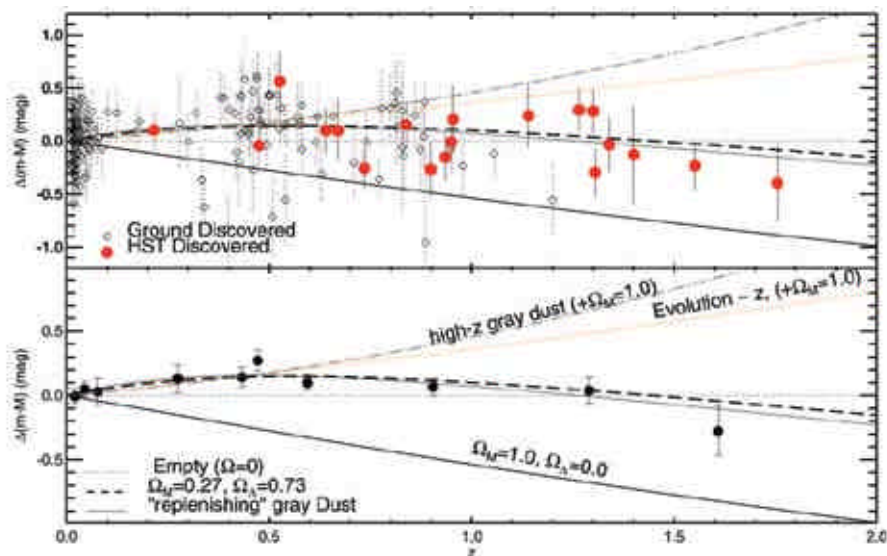


Figure 5. The residual Hubble diagram of SNe Ia relative to the expectation for an empty universe, compared to models; the bottom panel shows weighted averages in redshift bins²⁸.

expansion. However alternative explanations such as luminosity evolution proportional to lookback time, or extinction by dust which is maintained at a constant density are still possible. Moreover for reasons to do with how SNe Ia are detected, the dataset consists of approximately equal subsamples with redshifts above and below $z \sim 0.3$. It has been noted that this is also the redshift at which the acceleration is inferred to begin and that if these subsets are analysed *separately*, then the 142 ground-observed SNe Ia are consistent with deceleration; only when the 14 high- z SNe Ia observed by the *HST* are included is there a clear indication of acceleration²⁸. Clearly further observations are necessary particularly at the poorly sampled intermediate redshifts $z \sim 0.1-0.5$, as is being done by the *Supernova Legacy Survey*²⁹ and *ESSENCE*³⁰; there is also a proposed space mission – the *Supernova Acceleration Probe*³¹.

3 The spatial curvature and the matter density

Although the first indications for an accelerating universe from SNe Ia were rather tentative, the notion that dark energy dominates the universe became widely accepted rather quickly³². This was because of two independent lines of evidence which also suggested that there is a substantial cosmological constant. The first was that contemporaneous measurements of degree-scale angular fluctuations in the CMB by the *Boomerang*³³ and *MAXIMA*³⁴ experiments provided a measurement of the sound horizon (a ‘standard ruler’) at recombination³⁵ and thereby indicated that the curvature term $k \simeq 0$, i.e. the universe is spatially flat. The second was that, as had been recognized for some time, several types of observations indicate that the amount of matter which participates in gravitational clus-

tering is significantly less than the critical density, $\Omega_m \simeq 0.3$ (ref. 36). The cosmic sum rule then requires that there be some form of ‘dark energy’, unclustered on the largest spatial scales probed in the measurements of Ω_m , with an energy density of $1 - \Omega_m \simeq 0.7$. This was indeed consistent with the value of $\Omega_\Lambda \sim 0.7$ suggested by the SNe Ia data^{19,20} leading to the widespread identification of the dark energy with vacuum energy. In fact all data are consistent with $w = -1$, i.e. a cosmological constant, hence the model is termed Λ CDM (since the matter content must mostly be cold dark matter (CDM) given the constraint from BBN on the baryonic component).

Subsequently a major advance has come about with precision measurements of the CMB anisotropy by the *WMAP* satellite, and of the power spectrum of galaxy clustering by the *2dFGRS* and *SDSS* collaborations. The paradigm which these measurements test is that the early universe underwent a period of inflation which generated a Gaussian random field of small density fluctuations ($dr/r \sim 10^{-5}$ with a nearly scale-invariant ‘Harrison-Zeldovich’ spectrum: $P(k) \propto k^n$, $n \simeq 1$), and that these grew by gravitational instability in the sea of (dark) matter to create the large-scale structure (LSS) as well as leaving a characteristic anisotropy imprint on the CMB. The latter are generated through the oscillations induced when the close coupling between the baryon and photon fluids through Thomson scattering is suddenly reduced to zero as the universe turns neutral at $z \sim 1000$ (ref. 35). The amplitudes and positions of the resulting ‘acoustic peaks’ in the angular power spectrum of the CMB are sensitive to the cosmological parameters and it was recognized that precision measurements of CMB anisotropy can thus be used to determine these accurately³⁷. However in practice there are many ‘degeneracies’ in this exercise because of the ‘prior’ assumptions that have to be made³⁸. An useful

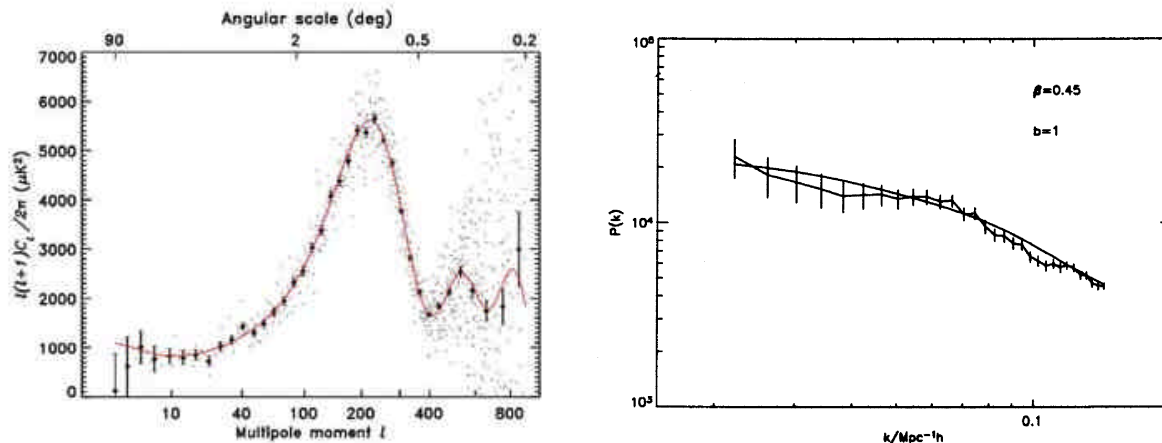


Figure 6. Angular power spectrum of the CMB measured by WMAP (grey dots are the unbinned data) and the fit to the Λ CDM model; the right panel shows the predicted matter power spectrum compared with the 2dFGRS data assuming no bias between galaxies and dark matter⁴⁰.

analogy is to see the generation of CMB anisotropy and the formation of LSS as a sort of cosmic scattering experiment, in which the primordial density perturbation is the ‘beam’, the universe itself is the ‘detector’ and its matter content is the ‘target’³⁹. In contrast to the situation in the laboratory, neither the properties of the beam, nor the parameters of the target or even of the detector are known – only the actual ‘interaction’ may be taken to be gravity. In practice, therefore, assumptions have to be made about the nature of the dark matter (e.g. ‘cold’ non-relativistic or ‘hot’ relativistic?) and about the nature of the primordial perturbation (e.g. adiabatic or isocurvature?) as well as its spectrum, together with further ‘priors’ (e.g. the curvature parameter k or the Hubble constant h) before the cosmological density parameters can be inferred from the data.

Nevertheless as Figure 6 shows, the angular spectrum of the all-sky map of the CMB by WMAP is in impressive agreement with the expectation for a flat Λ CDM model, assuming a power-law spectrum for the primordial (adiabatic only) perturbation⁴⁰. The fitted parameters are $\Omega_B h^2 = 0.024 \pm 0.001$, $\Omega_m h^2 = 0.14 \pm 0.02$, $h = 0.72 \pm 0.05$, with $n = 0.99 \pm 0.04$ so it appears that this does herald the dawn of ‘precision cosmology’. Even more impressive is that the prediction for the matter power spectrum (obtained by convoluting the primordial perturbation with the CDM ‘transfer function’) is in good agreement with the 2dFGRS measurement of the power spectrum of galaxy clustering⁴¹ if there is no ‘bias’ between the clustering of galaxies and of CDM. Subsequent studies using the power spectrum from SDSS⁴² and also from spectral observations of the Lyman- α ‘forest’ (intergalactic gas clouds)⁴³ have confirmed these conclusions and improved on the precision of the extracted parameters. Having established the consistency of the Λ CDM model, such analyses also provide tight constraints, e.g. on a ‘hot dark matter’ (HDM) component which translates into a bound on the summed neutrino masses of $\sum m_n < 0.42$ eV.

It must be pointed out, however, that cosmological models *without* dark energy can fit exactly the same data by making different assumptions for the ‘priors’. For example, an Einstein-deSitter model is still allowed if the Hubble parameter is as low as $h \simeq 0.46$ and the primordial spectrum is not scale-free but undergoes a change in slope from $n \simeq 1$ to $n \simeq 0.9$ at a wavenumber $k \simeq 0.01$ Mpc^{-1} (ref. 44). To satisfactorily fit the LSS power spectrum also *requires* that the matter not be pure CDM but have a HDM component of neutrinos with (approximately degenerate) mass 0.8 eV (i.e. $\sum m_n = 2.4$ eV) which contribute $\Omega_n \simeq 0.12$. An alternative to a sharp break in the spectrum is a ‘bump’ in the range $k \sim 0.01$ – 0.1 Mpc^{-1} , such as is expected in models of ‘multiple inflation’ based on supergravity⁴⁵. Although such models might appear contrived, it must be kept in mind that they do fit all the precision data (except the SNe Ia Hubble diagram) without dark energy and that the degree to which parameters must be adjusted pales into insignificance in comparison with the fine-tuning required of the cosmological constant in the Λ CDM model!

Conclusions

Thus for the moment we have a ‘cosmic concordance’ model with $\Omega_m \sim 0.3$, $\Omega_\Lambda \sim 0.7$ which is consistent with all astronomical data but has *no* explanation in terms of fundamental physics. One might hope to eventually find explanations for the dark matter (and baryonic) content of the universe in the context of physics beyond the Standard Model but there appears to be little prospect of doing so for the apparently dominant component of the universe – the dark energy which behaves as a cosmological constant. Cosmology has in the past been a data-starved science so it has been appropriate to consider only the simplest possible cosmological models in the framework of general relativity. However now that we are faced with this serious confrontation between fundamental physics and cosmology, it is surely appropriate to reconsider the basic assump-

tions (homogeneity, ideal fluids, trivial topology, ...) or even possible alternatives to general relativity.

General relativity has of course been extensively tested, albeit on relatively small scales. Nevertheless the standard cosmology based on it gives a successful account of observations back to the BBN era⁴⁶. However it is possible that the ferment of current theoretical ideas, especially concerning the possibility that gravity may propagate in more dimensions than matter, might suggest modifications to gravity which are significant in the cosmological context^{47,48}. Astronomers are, of course, entitled to, and will continue to, analyse their data in terms of well-established physics and treat the cosmological constant as just one among the parameters specifying a cosmological model. However it is important for it to be recognized that ‘Occam’s razor’ does not really apply to the construction of such models, given that there is no physical understanding of the key ingredient Λ .

Landau famously said ‘Cosmologists are often wrong, but never in doubt’. The situation today is perhaps better captured by Pauli’s enigmatic remark – the present interpretation of the data may be ‘... not even wrong’. However we are certainly not without doubt! The crisis posed by the recent astronomical observations is not one that confronts cosmology alone; it is the spectre that haunts any attempt to unite two of the most successful creations of 20th century physics – quantum field theory and general relativity. It is quite likely that the cosmological constant which Einstein allegedly called his ‘biggest blunder’ will turn out to be the catalyst for triggering a new revolution in physics in this century.

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