and when $Y > p_n^2$

$$P = 0.$$

Therefore, the above expression gives only twinprimes.

Remark 5. For any x in (p_n, p_n^2), the number of primes not more than x is given by

$$\Pi(x) = \frac{1}{4\pi}\int_0^{2\pi}\prod_{i=1}^{n}\frac{\sin((Q-Q_i)/2)\theta).\sin((n-1)Q\theta/2).\sin([x]\theta/2)\cos(([x]+1)\theta/2)}{\sin(Q_i\theta/2).\sin(Q\theta/2)\sin(\theta/2)} \times d\theta + n. \quad (20)$$

Noting that the angles in each set by the summation are in arithmetic progression, the above equation can be proved by summing over $a_n, a_{n-1}, a_{n-2}, \ldots, a_1$, from,

$$\Pi(x) - \Pi(p_n) = \frac{1}{4\pi}\int_0^{2\pi}\sum_{a_1=1}^{(p_1-1)}\cdots \sum_{a_n=1}^{(p_n-1)}\sum_{b=1}^{(n-1)}\sum_{t=1}^{[x]}\cos(\Sigma_1^n a_i Q_i - bQ)\theta\cos(t\theta)\,d\theta. \quad (21)$$

Remark 6. If $\Pi_2(x)$ represents the number of twinprimes not exceeding x; $p_n < x \leqslant p_n^2$, then,

$$\Pi_2(x) - \Pi_2(p_n) = \frac{1}{4\pi}\int_0^{2\pi}\sum_{a_1 \neq a_1'}\cdots \sum_{a_n \neq a_n'}\sum_{b=1}^{(n-1)}\sum_{t=1}^{[x]}\cos(\Sigma_1^n a_i Q_i - bQ)\theta.\cos(t\theta)\,d\theta. \quad (22)$$

Since all the twinprimes up to p_n are known, $\pi_2(p_n)$ is known.

$$\Pi_2(x) = \frac{1}{4\pi}\int_0^{2\pi}\sum_{a_1 \neq a_1'}\cdots \sum_{a_n \neq a_n'}\sum_{b=1}^{(n-1)}\sum_{t=1}^{[x]}\cos(\Sigma_1^n a_i Q_i - bQ)\theta\cos(t\theta)\,d\theta + \Pi_2(p_n). \quad (23)$$

The twinprime conjecture follows immediately if the right side (23) is greater than $\Pi_2(p_n)$

3. Detailed proofs for remarks 4 and 5

Detailed proofs for remarks 4 and 5 with numerical examples, application in connection with p_{n+1}, P, $\Pi(x)$ and $\Pi_2(x)$ will be furnished in a separate paper.

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