

Subject to $1 \leq a_i \leq (p_i - 1)$ for all i , we have secured $(m, Q) = 1$.

If in addition, the condition that $((m + 2), Q) = 1$ is to be satisfied, we should have

$$\left(\left(\sum_1^n a_i Q_i - \sum_1^n a'_i Q_i \right), Q \right) = 1. \tag{11}$$

That is to say $a_i \neq a'_i$ for each i . By a similar method, it is possible to ensure

$$\{[m(m + 2)(m + 6)], Q\} = 1 \tag{12}$$

and so on.

Remark 3. In Theorem 2, substituting $(-J^2)$ for (J^2) , we get

$$\sum_{a_1=1}^{(p_1-1)} \dots \sum_{b=1}^{(p_n-1)} \sum_{b=1}^n X^{-J^2} = 2(X^{-1} + X^{-p_{n+1}^2} + \dots) \tag{13}$$

where X is any given positive number.

Therefore,

$$\frac{1}{2}(\text{LHS}) - X^{-1} = M = (X^{-p_{n+1}^2} + X^{-p_{n+2}^2} + \dots) \tag{14}$$

where LHS stands for the expression on the left side of the equation.

Multiplying both sides of the above equation by $X^{p_{n+1}^2}$, we get

$$M \cdot X^{p_{n+1}^2} = 1 + X^{-p_{n+2}^2 + p_{n+1}^2} + \dots = 1 + R \text{ (say).}$$

For values of $X \geq 2$, it is clear that $R < 1$. Therefore,

$$1 < M \cdot X^{p_{n+1}^2} < 2 \tag{15}$$

Taking logarithms of both sides of (15), to the base X , we get

$$\begin{aligned} 0 < p_{n+1}^2 + \log_X M < 1 \\ p_{n+1}^2 > -\log_X M > p_{n+1}^2 - 1 \end{aligned}$$

That is to say, the integral part of $(-\log_X M)$ is equal to $(p_{n+1}^2 - 1)$, and,

$$p_{n+1}^2 = 1 + \left[-\log_X \left(\frac{1}{2} \sum_{a_1=1}^{(p_1-1)} \dots \sum_{a_n=1}^{(p_n-1)} \sum_{b=1}^n X^{-(\sum_1^n a_i Q_i - bQ)^2} - (1/X) \right) \right]. \tag{16}$$

Equation (16) is thus the formula for the $(n + 1)$ th prime.

P is the first twinprime in (p_n, p_n^2) if P differs from zero by the equation:

$$P^2 = Y^2 \left(1 - \left[\sin^2 \left(\frac{\pi}{2} - [2^{-(Y^2 - p_n^2)}] \right) \right] \right) \tag{17}$$

where Y is given by the below expression:

$$Y^2 = 1 + \left[-\log_X \left(\frac{1}{2} \sum_{a'_1 \neq a_1=1}^{(p_1-1)} \dots \sum_{a'_n \neq a_n=1}^{(p_n-1)} \sum_{b=1}^n X^{-(\sum_1^n a_i Q_i - bQ)^2} - (1/X) \right) \right]. \tag{18}$$

It could be noted that when $Y < p_n^2$

$$P = Y \tag{19}$$