

Formula for primes, twinprimes, number of primes and number of twinprimes

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Abstract. Formulae for computing the n th prime, twinprime, the number of primes smaller than a given integer, and the number of twinprimes smaller than a given integer are presented. Proofs for the development are also furnished.

Keywords. Primes; twinprimes

1. Introduction

Let $p_1 = 2, p_2 = 3, p_3 = 5, \dots, p_n$ be the sequence of the first n primes, and,

$$Q = \prod_{i=1}^n p_i \quad (1)$$

This paper presents formula to compute the $(n+1)$ th prime which is p_{n+1} and other allied results.

Let

$$Q_i = Q/p_i, \quad \text{for } i = 1, 2, 3, \dots, n \quad (2)$$

$$1 \leq a_i \leq (p_i - 1)$$

$$K = n - \sum_1^n (1/p_i) \quad (3)$$

$0 \leq b \leq [K]$, where $[K]$ denotes the integral part of K and

$$J = \sum_1^n a_i Q_i - bQ \quad (4)$$

The following four theorems are first proved.

THEOREM 1.

For n greater than 1,

$$\sum_{a_1=1}^{(p_1-1)} \dots \sum_{a_n=1}^{(p_n-1)} \sum_{0 \leq b \leq [K]} X^J = X + (1/X) + f(X) + f(1/X) \quad (5)$$

where $f(X) = \sum a_m X^m$

$$1 < m \leq [K]Q \prod_{i=1}^n (1 - (1/p_i)), \quad (m, Q) = 1, \text{ and}$$

$a_m = 1$ for all $m < Q$ and $(m, Q) = 1$ and $a_m = 0$ for some $m \geq Q$.